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# The formation of price expectations: a case study of the soybean market 

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## Langley, Suchada Vichitakul

THE FORMATION OF PRICE EXPECTATIONS: A CASE STUDY OF THE SOYBEAN MARKET
$\square$

# The formation of price expectations: A case study of the soybean market 

## by

Suchada Vichitakul Langley

# A Dissertation Submitted to the Graduate Faculty in Partial Fulfillment of the Requirements for the Degree of DOCTOR OF PHILOSOPHY 

Major: Economics

## Approved:

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## CHAPTER.1. INTRODUCTION

The purpose of this research is to build a model of decision-making behavior in the soybean market. The decision rules are derived from a model of dynamic optimization under uncertainty, which pepresentative soybean producers (farmers) and soybean processors have to solve.

A representative soybean producer has to make a decision as to how many acres of soybeans to plant before the price of soybeans for the new crop year is known. A decision also has to be made as to the inventory level of beans to be held on-farm at the end of each quarter to meet demand for feed, seed and for speculation. A representative processor has to make decisions as to the quantity of beans to crush into meal and oil to meet demand for meal and oil and the inventory level of beans to hold at the end of each quarter for transaction in the following quarter and for speculation. Both groups have to make predictions concerning the prices received for output and the prices paid for inputs.

The decision rules derived from the optimization process depend upon, among other things, future prices of beans. This research explores three regimes of price expectations. They are:

1. rational expectations;
2. adaptive expectations; and,
3. cash-futures price expectations.

Rational expectations require economic agents to have a structural model and to utilize all available information. This information forms an agent's constraint set. An agent's observed behavior will change if the
constraints change. The constraints include the laws of motion that describe the exogenous stochastic variables, such as prices paid for input factors, innovation, government policy variables and other related variables. Changes in agents' perceptions of the laws of motion will change their decision rules on choice variables.

Expectations are subjective, personal and not easily measured. Econometricians have used distributed lag procedures in attempting to capture expectations. Use of a lag distribution implies that an agent's best judgement about the future is captured in historical data; or it can be said that the future behaves like the past. In general, this approach to expectations performs well for the sample period; however, questions arise when one tries to use it to make forecasts beyond the sample period (ex ante forecasts). If the structure of the economy changes, econometricians cannat depict these changes in distributed lag models.

Rational expectations are an alternative. Formulation and estimation of the rational-expectations model are time consuming and costly. One has to make a trade-off between accuracy and cost. Many economists have suggested that due to the high cost of gathering and processing information, it is "economically rational expectations" to use adaptive expectations or distributed lag expectations. Thus, an adaptive expectations in which future prices being functions of lagged prices is the second model to be investigated in this research.

The other alternative is to make use of the futures market institution. It is generally believed that the price of the nearest futures
contract of a commodity is an unbiased estimate of the cash price at the maturity date of the contract. Some economists have used prices of contracts that mature at harvest time to forecast the cash price at harvest, arguing that futures prices fully reflect the information needed to formulate price expectations. If the futures prices are an unbiased estimate of the expected cash prices, it is, in fact, rational expectations as well.

The plan of this research is to briefly explain the nature of the soybean market, its products, and their price variability in Chapter two. Chapter three surveys theories and concepts of price expectations which will be used throughout this study. Formulation of the soybean model is in Chapter four, where three alternative models are built based upon different price expectation regimes. Though all three models have the same structure and objective functions, the way agents form their expectations differ, resulting in a different set of decision rules for each price-expectation regime. Chapter five presents the results of Granger (19691) causality tests and estimation of the model. Fuller's (1976) methodology on searching for a stationary series is extensively utilized. Most of the estimations are on the time domain except in some parts where the frequency domain seems appropriate. Estimation of the model under the three price-expectation regimes is reported. Chapter six contains the results of dynamic simulation of the Quasi-Rational model and suggestions for further research.

## CHAPTER 2. THE STRUCTURE OF THE SOYBEAN MARKET

Soybeans have been known for a thousand years as a food crop in Asia, particularly China. In the United States, soybean production began to expand significantly in the 1940s (Figure 2.1). This expansion was the result of a high U.S. tariff against imported tropical oil seeds, expansion of livestock production after World War II, government help, and achievement in research efforts directed toward increasing the uses of soybeans. Soybean meal, one of its two main products, is a high protein feed for poultry and 1ivestock. Soybean oil, the other main product, is a major oil used in household products such as margarine, cooking oils, and salad oils. Soybean oil also has potential to be a substitute for diesel fuel (Levins and Meyers, 1981). Figure 2.2 illustrates the utilization of soybeans.

During the crop year, farmers may sell beans or hold them in on-farm storage. Over fifty percent of beans sold go to the processing industry; the rest are exported and/or held in off-farm inventory.

## Production of Soybeans

The crop year (marketing year) for U.S. soybean production is September to August. Farmers make production decisions and plant soybeans In the third quarter of the crop year (March-May). Acreage planted in soybeans depends upon, among other things, the price farmers expect to receive ( $P_{t}$ ) when new production is realized. Heady and Rao (1967) studied an acreage response function for soybeans and found that the price ratio of soybeans to corn and the yield ratios of soybean-oats and soybean-corn are


Figure 2.1 Acreage planted of Soybean, soybean production, crush and export


Figure 2.2 Domestic soybean utilization, (Houck, Ryan, and Subotnik, 1972)
significant explanatory variables. They also found that July rainfall is significant for soybean production. Houck, Ryan and Subotnik (1972) estimated acreage harvested for U.S. major soybean-producing regions as a function of, among other things, the expected prices of soybeans and its competing commodities. They used one-year lags of actual prices as proxy variables for expected prices. Meyers and Hacklander (1979) included price ratios of soybean-corn, soybean-cotton and the ratio of corn-soybean price support in their acreage-planted equation for next year's crop.

Acreage planted is a decision rule to be derived from the optimal behavior presented in Chapter four. Soybean production is approximately obtained from a technical relationship of the multiplication of soybean yield per harvested acre by acres planted and an adjustment factor. The other important decision made by farmers is the amount of soybean inventory to hold on-farm. Most of the literature has neglected this issue. The reason is that on-farm inventory accounted for only a small portion of total inventory in the past. Although the ending year soybean inventory on-farm is small, the figures are quite different during the year, especially from quarter to quarter (Figure 2.3). The movement of soybean inventory on-farm displays strong seasonality. On an average, during 1958 through 1979 farmers held 198.0 million bushels of beans on-farm, although the minimum and maximum level ranged from zero to 893 million bushesl. It is also significant that the mean level has moved up to 305 million bushels during the 1970s. The decision behavior on holding beans on-farm is

derived in Chapter four. The results from this investigation can be compared to the behavior of soybean processors in holding beans off-farm.

The Soybean Processing Industry
The soybean processing industry became large and complex after World War II. As shown in Figure 2.2, the two main products of soybeans are soybean meal and oil. Both of these products are important in their own way. On the average, 10.7 lbs . of oil and 48 lbs . of meal can be extracted from one bushel of beans. Oil yield fluctuates moderately from year to year due to the oil content of beans, which depends mainly upon weather. Meal yield is relatively stable. Soybean processors demand an even flow of beans to supply their crushing facilities. In general, processors will hold a large amount of beans during the immediate postharvest period (Figure 2.4). A peak in processing generally occurs in November and December, when there is a large flow of beans from farms. Thus, soybean processors have to make decisions concerning the quantity of beans to crush in order to supply current meal and oil consumption and the quantity of beans to hold in inventory in order to guarantee adequate supply of beans for crushing throughout the year. As shown in Table 2.1, about half of annual soybean production is crushed. The remaining portion is either exported or held in inventory. The demand for soybean crush depends upon soybean price, value of crushing products, and crushing capacity (Meyers and Hacklander, 1979). In this research, soybean crush and holding inventory of beans off-farm are decision rules which a representative soybean processor has to solve.


Table 2.1. Soybeans, soymeal and soyoil: supply, disappearance and their distributions in marketing year 1979/80 and 1980/81

|  | Units | 1979780 | 1980/81 |
| :---: | :---: | :---: | :---: |
| Soybeans |  |  |  |
| Production | Mil. bu. | 2,268 | 1,792 |
| Beginning stocks | " | 174 | 359 |
| Ending stocks | " | 359 | 320 |
| Crushing | " | 1,123 | 1,020 |
| Exports | " | 875 | 724 |
| Feed, seed | " | 68 | 66 |
| Average farm price | \$/bu. | 6.28 | 7.57 |
| Soybean meal |  |  |  |
| Production | 1000 s. tons | 27.105 | 24,312 |
| Beginning stk. | " | 267 | 226 |
| Domestic disappearance (include shipments) | 11 | 19,215 | 17,597 |
| Exports |  | 7,932 | 6,778 |
| Ending stk. |  | 226 | 163 |
| Decatur price | \$/ton | 181.9 | 218.18 |
| Soybean oil |  |  |  |
| Production | Mil. 1b. | 12,105 | 11,270 |
| Beginning stk. |  | 776 | 1,210 |
| Ending stk. |  | 1,210 | 1,736 |
| Domestic disappearance - (include shipments to |  |  |  |
| U.S. territories |  | 8,981 | 9,115 |
| Exports <br> (exclude shipments) | Mil. 1b. | 2,690 | 1,629 |
| Decatur price | ¢/1b. | 24.3 | 22.7 |

Soybean Market, Its Channels and Its Price
After harvest, farmers may sell their beans or hold them on the farm. In general, most beans are moved to local elevators. $P_{t}$ is the price farmers receive from local elevators, who, in turn, act as arbitragers in handling beans to soybean processors, exporters or larger elevators. Elevators take advantage of the difference between soybean price received by farmers ( $\mathrm{P}_{\mathrm{t}}$ ) and wholesale price of beans ( $\mathrm{PS}_{\mathrm{t}}$ ), as showing in Figure 2.5. In this research we use the Decatur, Illinois, price to represent the U.S. wholesale price for soybeans. Figure 2.6 illustrates the relationship between soybeans and their products. Soybean meal and soybean oil quantities are shown in beans equivalent. Soybeans, soybean meal, and soybean oil markets are shown in panels (a)-(d), (e)-(h) and (i)-(1) respectively. Given demand for soybean crush, exports, and inventories, we can derive total demand by summing them up. Total demand and supply of beans will determine the price of beans (PS). At a quantity of soybean crush, given a price of beans (PS), total production of meal and oil are determined. Given domestic demand, export demand and inventories for meal and oil, the prices of soybean meal and soybean oil are determined. The demand for soybeans crushed, inventories and acreage planted are derived from optimization procedures which will be described in Chapter four. Exports are exogenous to this study. The total demand for beans and supply of beans thus in turn would determine the price of beans. By keeping the model manageable and simple, some insights may be drawn from this study.



CHAPTER 3: THEORETICAL SURVEY ON THE ECONOMICS OF EXPECTATIONS

As Hayek (1945) said, "...the problem of a rational economic order is determined precisely by the fact that the knowledge of the circumstance of which we must make use never exists in concentrated or integrated form, but solely as the dispersed bits of incomplete and frequently contradictory knowledge which all the separate individuals possess." The economic problem is a problem of how to secure the best use of resources. It is a problem of the utilization of knowledge. Economists are interested in expectations -- what kind of information is used and how it is put together to make predictions about the future. Rational expectations in the sense of Muth (1961) states that economic agents form their expectations as if they know the process which will generate the actual outcomes. Or we can say that people's subjective probability distributions describing future outcomes are identical to the corresponding objective probability distribution conditional on the true model of the economy. Descriptively, the expectations which are formed to predict future events will be the same as the predictions of the relevant economic theory. Economic agents have perceptions that their forecasts will be correct. What is missing here is that this hypothesis does not say how economic agents derive the knowledge which they would use to formulate expectations. To integrate the learning process or information formation into rational expectations is a challenging task. As Rawls (1971) said, "...the rationality of a person's choice does not depend upon how much he knows, but only upon how well he reasons from whatever information he has, however incomplete. Our decision is
perfectly rational provided that we face up to our circumstances and do the best we can." Thus, for Rawls, "rational expectations" means economic agents use optimally whatever information is available.

Rational expectations analysis has played a crucial role in recent research in stochastic dynamics and control. In contrast to rational expectations, adaptive expectations is based upon the idea that the expected value of a variable is a fixed weighted average of past observations of that variable. The forecasts under an adaptive model assume that the value of a variable will behave in the future as it did in the past. This assumption is rigid, and it does not reflect any changes which may occur. Rational expectations, however, are more responsive to changes in economic variables. Even though the idea of rational expectations is more conceptually sound, a drawback still exists in the formulation and estimation of a rational expectations model. It is a challenging area of research.

In this section, we present a selective survey of research on anticipatory commodity prices (especially for stored commodities), the rational expectations hypothesis, the adaptive price expectations hypothesis, and the cash-future price relationship. Finally, the causality hypothesis is summarized.

## Anticipatory Commodity Prices

The work of Working (1958) improved understanding of the nature of commodity price fluctuation. He explained that if prices are formed in a free market, they will be formed under the influence of expectations. Commodities, such as soybeans, are produced once a season. Consumers are
forced to anticipate their wants during all the future months of the season, to guess at the prices which they will later have to pay for their consumptions, and to forecast income they will have during the season. Assuming supply is fixed, expectations are involved only in the formation of demand. Working's concept of demand is not a schedule of amounts that are bought during any particular interval of time, but is a schedule of amounts that are held at a particular time. The holding schedule is based on expectations concerning consumption demand and the existing supply conditions. The model Working proposes has the following characteristic:
(1) It assumes prices to be formed through the medium of human decision, on the basis of information realistically available to traders.
(2) It assumes the existence of conditions, within and around the market, such as have actually prevailed in the world during recent years.
(3) It assumes there are a large number of traders.

Information available to traders is sometimes incomplete, false or erroneous. Nearly all of the traders are persons of rather exceptional trading ability and judgement, emotionally stable, with knowledge and skill, and they keep informed on their business. Thus, traders must seek information to guide their actions in price formation. They seek information on the prospective supplies, consumption, changes in business conditions, and in the general price level. They may seek to obtain information in advance of routine publication of the information.

## Selective Review of Rational Expectations

Economic agents, in general, react to current and anticipated future information. However, it is controversial how their anticipations are
formed. Muth (1961) bases his theory of rational expectations on three things, "...(1) Information is scarce, and the economic system generally does not waste it; (2) The way expectations are formed depends specifically on the structure of the relevant system describing the economy; and, (3) A 'public prediction,' in the sense of Grunberg and Modigliani..., will have no substantial effect on the operation of the economic system (unless it is based on inside information)." Thus, expectations in the sense of Muth are based upon costless information. In order to derive the price expected to prevail at the $t^{\text {th }}$ period on the basis of information through ( $t-1)^{\text {th }}$ period ( $\left(-1 P_{t}^{e}\right.$ ), Muth assumes "...(1) The random disturbances are normally distributed; (2) Certainty equivalents exist for the variables to be predicted; and, (3) The equations of the system, including the expectations formulas, are linear."

If the expectations which are formed to predict future events are the same as the predictions of relevant economic theory, then these expectations are rational. Mathematically, the expected value of market price at time $t$ equals the market price expected to prevail during the $t^{\text {th }}$ period on the basis of information available at time $t-1$, or,

$$
\begin{equation*}
E_{t-1}\left(P_{t} \mid \Omega_{t-1}\right)={ }_{t-1} P_{t}^{e} \tag{3.1}
\end{equation*}
$$

where $E_{t-1}=$ expected value operator based on information available through the $(t-1)^{\text {th }}$ period,
$P_{t}$. $=$ the market price at $t$,
$t-1 P_{t}^{e}=$ the expected price to prevail at $t$ on the basis of information through t-1.
$\Omega_{t-1}=$ information set available at time $t-1$.
The following is Muth's (1961) model:
$\left.\begin{array}{ll}\text { Demand: } & C_{t}=-\beta P_{t}, \quad \beta>0 \\ \text { Supply: } & \dot{Y}_{t}=\gamma\left({ }_{t-1} P_{t}^{e}\right)+\mu_{t}, \gamma>0 \\ \text { Market Clearing: } & Y_{t}=C_{t}\end{array}\right\}$
where $C_{t}=$ the amount consumed,
$Y_{t}=$ units produced in a period lasting as long as the production lag, and,
$\mu_{t}=$ stochastic disturbance, e.g. variations in yields due to weather.

All variables are deviations from equilibrium values. Solving (3.2) for $P_{t}$, we obtain:

$$
\begin{equation*}
P_{t}=-\frac{\gamma}{\beta}\left(t-1 P_{t}^{e}\right)-\frac{1}{\beta} \mu_{t} \tag{3.3}
\end{equation*}
$$

If there is no serial correlation of $\mu_{t}$, and

$$
\begin{align*}
& E_{t-1}\left(\mu_{t}\right)=0 \text {, then we obtain } \\
& E_{t-1}\left(P_{t}\right)=-\frac{\gamma}{\beta} \cdot\left(t-1 P_{t}^{e}\right) \tag{3.4}
\end{align*}
$$

Using rational expectations (3.1), then ( ${ }_{t-1} P_{t}^{e}$ ) $=0$, or, the price expected to prevail at $t$ equals the equilibrium price.

In the case of serial correlation in $\mu_{t}$, Muth assumes that $\mu_{t}$ has the following representation:

$$
\begin{equation*}
\mu_{t}=\sum_{i=0}^{\infty} w_{i} \varepsilon_{t-i} \tag{3.5}
\end{equation*}
$$

and $\quad E\left(\varepsilon_{j}\right)=0 \quad E\left(\varepsilon_{i} \varepsilon_{j}\right)= \begin{cases}\sigma^{2} & \text { if } i=j \\ 0 & \text { if } i \neq j\end{cases}$
Thus, $E\left(\mu_{t}\right)=\sum_{i=1}^{\infty} w_{i} \varepsilon_{t-i}$ where $E\left(\varepsilon_{t}\right)=0$. Taking expected values of (3.3) conditioning on $\Omega_{t-1}$, we get

$$
\begin{align*}
& t-1 P_{t}^{e}\left(1+\frac{\gamma}{\beta}\right)=-\frac{1}{\beta} \sum_{i=1}^{\infty} w_{i} \varepsilon_{t-i} \\
& t-1 P_{t}^{e}=-\frac{1}{\gamma+\beta} \sum_{i=1}^{\infty} w_{i} t-i \tag{3.6}
\end{align*}
$$

Substituting (3.5) and (3.6) into (3.3), we get

$$
\begin{align*}
& P_{t}=+\frac{\gamma}{\beta}\left(\frac{1}{\gamma+\beta}\right) \sum_{i=1}^{\infty} w_{i} \varepsilon_{t-i}-\frac{1}{\beta} \sum_{i=0}^{\infty} w_{i} \varepsilon_{t-i} \\
&=\frac{\gamma}{\beta}\left(\frac{1}{\gamma+\beta}\right) \sum_{i=1}^{\infty} w_{i} \varepsilon_{t-i}-\frac{1}{\beta} w_{0} \varepsilon_{t}-\frac{1}{\beta} \sum_{i=1}^{\infty} w_{i} \varepsilon_{t-1} \\
&=\left[\frac{\gamma}{\beta}\left(\frac{1}{\gamma+\beta}\right)-\frac{1}{\beta}\right] \sum_{i=1}^{\infty} w_{i} \varepsilon_{t-i}-\frac{1}{\beta} w_{0} \varepsilon_{t} \\
&=\frac{1}{\beta}\left(\frac{\gamma}{\gamma+\beta}-1\right) \sum_{i=1}^{\infty} w_{i} \varepsilon_{t-1}-\frac{1}{\beta} w_{o} \varepsilon_{t} \\
& P_{t}=-\left(\frac{1}{\gamma+\beta}\right) \sum_{i=1}^{\infty} w_{i} \varepsilon_{t-i}-\frac{1}{\beta} w_{o} \varepsilon_{t} \tag{3.7}
\end{align*}
$$

or we can write $\mathrm{P}_{\mathrm{t}}$ as:

$$
\begin{equation*}
P_{t}=\sum_{i=0}^{\infty} W_{i} \varepsilon_{t-1} \tag{3.8}
\end{equation*}
$$

If the following conditions hold,

$$
\begin{gather*}
W_{0}=-\frac{1}{\beta} w_{0} \text { and } \\
W_{i}=-\frac{1}{\gamma+\beta} w_{i} ; \text { for } i=1,2,3 \ldots \\
\text { and } t-1 P_{t}^{e}=\sum_{i=1}^{\infty} W_{i} \varepsilon_{t-i} \tag{3.9}
\end{gather*}
$$

Thus, the expected price under this hypothesis depends upon the restrictions imposed on the parameters of the structural model and the stochastic term. Muth also proves that we can write $t_{t-1} \mathrm{P}_{\mathrm{t}}^{\mathrm{e}}$ as the following:

$$
\begin{equation*}
t-1 P_{t}^{e}=\sum_{j=1}^{\infty} V_{j} W_{i-j} \tag{3.10}
\end{equation*}
$$

However, for (3.9) to equal (3.10), the following restrictions are needed:

$$
\begin{align*}
W_{i} & =\sum_{j=1}^{1} V_{j} W_{i-j}  \tag{3.11}\\
\text { for } 1 & =1,2,3 \ldots
\end{align*}
$$

The exact form of $t-1 P_{t}^{e}$ depends upon $\beta, \gamma$ and $w_{i}$. If we assume that all $w_{i}=1$ for $i=0,1,2 \ldots$, then

$$
\begin{gather*}
W_{0}=-1 / \beta \text { and } W_{i}=-1 /(\beta+\gamma) \text { and } t-1 P_{t}^{e} \text { is: } \\
t-1 P_{t}^{e}=\frac{\beta}{\gamma} \sum_{j=1}^{\infty}\left(\frac{1}{1+\frac{\beta}{\gamma}}\right)^{j} P_{t-j} \tag{3.12}
\end{gather*}
$$

The coefficient of (3.12) is a function of parameters $\beta$ and $\gamma$ in the structural model. We will see later that the appearance of (3.12) is similar to the adaptive expectation formation, but the interpretation of the two hypotheses are quite different.

Policy Implication of the Rational Expectation Hypothesis
Robert Lucas (1976) has pointed out that most econometric models which perform well in short-run forecasting provide no useful information as to the actual consequences of alternative economic policies. This argument is based upon the difference between the prior "true" structure and the. "true" structure after policy changes. Parameters in most econometric models are assumed to be invariant with respect to changes in economic policy. This may be an erroneous assumption.

Consider an economy which is characterized by the equation

$$
\begin{equation*}
Y_{t+1}=F\left(Y_{t}, X_{t}, \theta, \varepsilon_{t}\right) \tag{3.13}
\end{equation*}
$$

where $Y_{t}, X_{t}, \theta$ and $\varepsilon_{t}$ are a vector of state variables, exogenous variables, parameters, and random shocks, respectively. In econometric practices, one would estimate the values of the vector $\theta$, with $F$ being specified in advance. In fact, there is no presumption that ( $F, \theta$ ) will be easy to discover. Even when $\theta$ is approximately known by estimation, it is unlikely that it will remain stable under arbitrary change in $X_{t}$ such as a change in government policy variables. This is known as a case of "parametric drift." A well-known example is the Phillips Curve argument. If the changes in $\theta$ induced by policy changes is slow, then one may get good forecasts for a few periods; but, this is certainly too much to ask for in a dynamic society like the United States. If $X_{t}=G\left(Y_{t}, \alpha, \eta_{t}\right)$ where $G$ is known, $\alpha$ is a fixed parameter vector and $\eta_{t}$ is a vector of shocks, then $\theta$ also is a function of $\alpha$, i.e., $\theta(\alpha)$, which econometricians have to estimate.

A change in a government policy is viewed as a change in $\alpha$. Thus, a change in $\alpha$ will affect the system through $X_{t}$ and $\theta(\alpha)$. It is clear that to forecast the result of structural changes induced by policy changes, one must know $\theta(\alpha)$. These arguments are crucial to the proposed model under the rational expectations hypothesis in Chapter four. The detail of this policy implication will be discussed later.

## Formulating and Estimating Rational Expectations Mode1

We now turn to the econometric problems involved in dealing with the formulation of the rational expectations model. As mentioned earlier, Lucas (1976) has criticized current macroeconometric models for being invalid to any policy evaluation regardless of how well they perform over the sample period of in ex ante short-run forecasting because they fail to take into account how a change in policy affects the structure of the models.

Hansen and Sargent (1980) present a methodology for formulating and estimating rational expectations models. They use a simple one-factor employment model where a firm chooses a contingency plan for employment $n_{t}$ to maximize its expected present value subject to $n_{t-1}$. The firm maximizes:

$$
\begin{align*}
& \lim _{N \rightarrow \infty} E_{t} \sum_{j=0}^{N} \beta^{j}\left[\left(\gamma_{0}+a_{t+j}-w_{t+j}\right) n_{t+j}-\left(\frac{\gamma_{1}}{2}\right) n_{t+j}^{2}\right. \\
&\left.-(\delta / 2)\left(n_{t+j}-n_{t+j-1}\right)^{2}\right] \tag{3.14}
\end{align*}
$$

where $w_{t}$ is the real factor rental rate, and $a_{t}$ is a random shock to technology which is seen by the firm but unobserved by econometricians.
$\gamma_{0}, \gamma_{1}$ and $\delta$ are positive constants, and the discount factor $\beta$ is between zero and one. It is assumed that a stochastic process for $a_{t}$ can be discovered by the firm. This assumption is crucial for obtaining the decision rule (demand function) of $n_{t}$. The firm's decision rule contains the future value of the stochastic process $w_{t}$ which is to be predicted but
cannot be controlled. Granger-causality is used in order to search for variables which help to predict $w_{t}$. That is, any variables in the information set $\Omega_{t}$ which help to predict $w_{t}$ must appear in the decision rule $n_{t}, n_{t}$ must fail to Granger-cause $w_{t}$ for $w_{t}$ (and other variables which Granger-cause $w_{t}$ ) to be strictly exogenous to $n_{t}$. Hansen and Sargent derive a closed form for the decision rule $n_{t}$ in which $n_{t}$ is a function of its own lagged value $n_{t-1}$, current and ( $r-1$ ) lagged values of $x_{t}$ (where $x_{t}$ is contained within $w_{t}$ and other variables which Granger-cause $w_{t}$ ) and current and ( $q-1$ ) lagged values of $a_{t}$.
$X_{t}$ and $a_{t}$ have $r^{\text {th }}$ and $q^{\text {th }}$ order univariate autoregressive representation. The decision rule $n_{t}$ expresses the restrictions imposed across the decision rule and the parameters of the stochastic process for $X_{t}$ and $a_{t}$. More clearly, the firm's decision rule for $n_{t}$ is:

$$
\begin{equation*}
n_{t}=\rho_{1} n_{t-1}+\mu(L) x_{t}+\pi(L) a_{t} \tag{3.15}
\end{equation*}
$$

where

$$
\begin{gather*}
X_{t}=\varepsilon_{1} X_{t-1}+\varepsilon_{2} X_{t-2}+\ldots \varepsilon_{r} X_{t-r}+v_{t}^{x} \text {, or } \\
\varepsilon(L) X_{t}=v_{t}^{x}  \tag{3.16}\\
a_{t}=\alpha_{1} a_{t-1}+\alpha_{2} a_{t-2}+\ldots \alpha_{q} a_{t-q}+v_{t}^{a} \text {, or, } \\
\alpha(L)_{a_{t}}=v_{t}^{a}  \tag{3.17}\\
\mu(L)=-\frac{\rho_{1}}{\delta} U \varepsilon(\lambda)^{-1}\left[I+\sum_{j=1}^{r-1}\left(\underset{k=j+1}{\sum_{i}^{r} \lambda^{k-j}} \varepsilon_{k}\right) L^{j}\right], \text { and, }
\end{gather*}
$$

$$
\pi(L)=\frac{\rho_{1}}{\delta} \alpha(\lambda)^{-1}\left[I+\sum_{j=1}^{q-1}\left(\sum_{k=j+1}^{q} \lambda^{k-j} \alpha_{k}\right) L^{j}\right]
$$

U is a row vector with one being the first element and zero otherwise, and

I is an identity matrix.
The derivation of equation (3.15) is present in Appendix $A$. The existence of restrictions across parameters in the decision rule for $n_{t}$ and stochastic processes of $w_{t}$ and $a_{t}$ are important in the rational expectations model, as Lucas (1976) pointed out.

To estimate the parameters in the model, Hansen and Sargent solve $n_{t}$ as a function of current and past $X$ 's and $a^{\prime} s$. Quasi-maximum likelihood estimation is used to get asymtotic properties of maximum likelihood estimates.

The soybean model under this rational expectations hypothesis is 'presented in Chapter IV. 'This model is based upon Muth (1961), Hansen and Sargent (1980), and Sargent (1980).

## Adaptive Price Expectations

Taylor (1975) argued that if it takes time for agents to learn the exact nature of monetary policy (or any kind of policy), then the rational expectations technique might not be relevant for studying the immediate effects of a sudden shift in policy. During the transitory period, the rational expectations hypothesis is invalid. A learning process is
necessary. Rational expectations may have advantages for longer time horizons. Feige and Pearce (1976) also argue that there is a middle ground between autoregressive expectation and rational expectation, a so-called "economically rational expectations." Their concept emphasizes that economic agents should consider the trade-off between benefits and costs of added information when forecasting and anticipating, say, future inflation rates. They use nonnegligible cost information sets which serve as leading Indicators or information sets which satisfy Granger causality. When the costs of gathering and processing information are considered, autoregressive expectation or adaptive expectation models may be economically as sound as the rational expectation model.

A distributed lag model has been used to measure expectations for a long time. The general model is:

$$
\begin{equation*}
p_{t}^{e}=\sum_{i=0}^{\infty} v_{i} p_{t-i} \tag{3.18}
\end{equation*}
$$

where $p_{t}^{e}=$ the expected price at time $t$ formed at $t-1$. Economic agents form their expectations and thus make forecasts based entirely on the past history of prices. The issues involved in this hypothesis are the number of lags needed, serial correlation and the weights ( $v^{\prime}$ s) in the model. Distributed lag models which have been used in economic literature are static expectations, extrapolative expectations and adaptive expectations.

Static expectations have the form:

$$
\begin{equation*}
p_{t}^{e}=p_{t-1} \tag{3.19}
\end{equation*}
$$

Economic agents have the perception that the current expected price is the same as last period's actual price. It will give good forecasts if the price series follows a random walk.
"The Cobweb Theorem" by Ezekiel (1938) is a famous example of static price expectations. The cobweb theorem postulates that anticipated prices are current prices at the time of the production decisions. Thus, if a decision is made to produce at $t-1$ for output at $t$, then the expected price in period $t-1$ for period $t, p_{t}^{e}$, is given by the observed price at t-1.

Another form of price formation is the so-called extrapolative expectations. The expected price for period $t$ at period $t-1, p_{t}^{e}$, is:

$$
\begin{equation*}
P_{t}^{e}=P_{t-1}+\eta\left(P_{t-1}-P_{t-2}\right) \tag{3.20}
\end{equation*}
$$

$P_{t-1}$ and $P_{t-2}$ are observed prices in period $t-1$ and $t-2$. $\eta$ is Metzler's coefficient of expectation (Metzler, 1941). This price expectation is the modification of the static expectations (3.19). The purpose of (3.20) is only to take into consideration the most recent trend in prices.

Another form of price expectations is the adaptive expectations:

$$
\begin{equation*}
P_{t}^{e}=P_{t-1}^{e}+a\left(P_{t-1}-P_{t-1}^{e}\right) \tag{3.21}
\end{equation*}
$$

Expectations of price are based upon price expectations in the last period and the difference between the actual and the expected price in the last
period. The value of "a" represents the perception which economic agents believe about the direction of the expected price. The term "a" has been known as "the coefficient of expectations."

Using the lagged operator " $L$ ", $P_{t}^{e}$ can be written as:

$$
\begin{gather*}
P_{t}^{e}=\frac{1-\beta}{1-\beta L} P_{t-1} \\
\text { where } \beta=(1-a) \\
\text { Or } P_{t}^{e}=a \sum_{i=0}^{\infty}(1-a)^{i_{P}}{ }_{t-1-i} ; 0<a<1 \tag{3.22}
\end{gather*}
$$

The price expected to prevail at $t$, given the history of the market price, is the sum of the weighted average of all past prices.

It is helpful at this point to compare the expected prices $P_{t}^{e}$ in (3.12), (3.19), (3.20) and (3.22), as in Table 3.1.

1

$$
\begin{aligned}
& P_{t}^{e}-P_{t-1}^{e}+a P_{t-1}^{e}=a P_{t-1} \\
& P_{t}^{e}-(1-a) P_{t-1}^{e}=a P_{t-1} \\
& (1-(1-a) L) P_{t}^{e}=a P_{t-1} \\
& \text { where }(1-a)=\beta \text { or }(1-\beta)=a \text {, then we get } \\
& \qquad(1-\beta L) P_{t}^{e}=(1-\beta) P_{t-1} \\
& P_{t}^{e}=\frac{1-\beta}{1-\beta L} P_{t-1}
\end{aligned}
$$

Table 3.1. Comparison of expected prices

| Expectations | $\mathrm{P}_{t}^{\mathrm{e}}$ | Derivation from structural model |
| :---: | :---: | :---: |
| Static | $\mathrm{P}_{\mathrm{t}-1}$ | no |
| Extrapolative | $P_{t-1}+n\left(P_{t-1}-P_{t-2}\right) ; n>0$ | no |
| Adaptive | $a \sum_{i=0}^{\infty}(1-a)^{i_{P}}{ }_{t-1-1} ; 0<a<1$ | no |
| Rational | $\left(\frac{\beta}{\gamma}\right) \sum_{j=1}^{\infty}\left(\frac{1}{1+\left(\frac{\beta}{\gamma}\right)}\right)^{j} P_{t-j}$ | yes |

The first three price expectations have not been derived from structural models - as in the case of the rational expectations. Though the adaptive and rational price expectations have the same appearances, they have different interpretations. The coefficient " $a$ " is ad hoc, while " $\frac{\beta}{\gamma}$ " is tied to economic behavior of the model. The same argument holds for "(1-a)" and " $\left(\frac{1}{1+\left(\frac{\beta}{\gamma}\right)}\right)^{\prime \prime}$. Thus, any changes on structural parameters " $\beta$ " and " $\gamma$ " will certainly affect $p_{t}^{e}$ under rational expectations. Such changes on " $\beta$ " and " $\gamma$ " have no effect on $p_{t}^{e}$ under adaptive expectations.

Cash-Futures Price Expectations
The futures market is an organized market which facilitates trade among traders. It is used for hedging and speculating. The futures market performs various functions, such as providing price signals to participants concerning allocation of input use, production and consumption. If futures prices (near and/or more distant futures prices) respond to changes in market expectations, such as anticipated cash prices, then one may use (near) futures prices as a predictor for anticipated cash prices.

One may ask questions such as why cash and futures prices should be related, or, if they are related, what is the nature of their relationships? Had the correlation between them been small, hedging would be infeasible.

An expectations hypothesis
One of Samuelson's (1965) hypotheses on anticipated prices is:

$$
\begin{equation*}
y_{\theta, t}=E\left(P_{\theta} \mid P_{t}, P_{t-1} \ldots\right) ; \theta>t \tag{3.23}
\end{equation*}
$$

where $y_{\theta, t}$ is a futures price quote at $t$ for delivery time at $\theta$, and $P_{\theta}$ is the cash price to prevail at time of delivery $\theta$. Equation (3.23) says that the futures price quotes at $t$ for delivery time at $\theta$ is the expected cash price to prevail at time $\theta$, given current and historical cash prices. One of the problems facing many researchers is which futures contracts should be used. Tomek and Gray (1970) have tested whether future price quotes at the spring time (April) for the harvest time (November) contract are a reasonable forecast for cash price at harvest time for corn and soybeans.

The equation they test is:

$$
\begin{equation*}
P_{c}=a+b P_{f} \tag{3.24}
\end{equation*}
$$

where $P_{c}=$ cash price at harvest, and
$P_{f}=$ spring-time futures price for the harvest time contract. They statistically accept the hypothesis that "a" and "b" are zero and one, respectively. The acceptance of Tomek and Gray's hypothesis implies that there is a strong correlation between $P_{c}$ and $P_{f}$ and that $P_{f}$ must lead (in a statistical sense) $P_{c}$. Labys and Granger (1970, pp. 108-109) perform lead-lag tests for many commodities and use various contract futures prices. They found no strong evidence for the existence of such leads. They conclude that futures prices will not be useful for predicting cash prices. In the proposed model in Chapter IV, we would hypothesize that (3.23) is true.

## Causality Hypothesis

Granger (1969) has investigated causality between stochastic variables $X$ and $Y$ in a testable fashion. The basic assumption is that the future cannot cause the past. If $U$ is all of the information in the universe accumulated up to time $t-1$ and (U-Y) is all information apart from the specified series $Y$, then we can define causality as the following:

$$
Y \text { is causing } X \text {, denoted by } Y \rightarrow X \text {, if }
$$

$$
\begin{equation*}
\sigma^{2}(X \mid U)<\sigma^{2}(X \mid \bar{U}-\bar{Y}) \tag{3.25}
\end{equation*}
$$

where $\sigma^{2}(X / U)$ is the variance of the forecast of $X$ given all information in U. Equation (3.25) says we are better able to predict $X$ using all
available information than if the information apart from $Y(U-Y)$ has been used. This definition is different from feedback, instantaneous causality and causality lag. Feedback is said to occur when $Y$ is causing $X$ and $X$ is causing $Y$, denoted by $Y \leftrightarrow X$. Granger uses the following definition for feedback:

$$
\left.\begin{array}{l}
\sigma^{2}(X \mid \bar{U})<\sigma^{2}(X \mid \bar{U}-\bar{Y}),  \tag{3.26}\\
\sigma^{2}(Y \mid \bar{U})<\sigma^{2}(\bar{Y} \mid \bar{U}-\bar{X})
\end{array}\right\}
$$

where the one line above $U$ or $U-Y$ refers to past information excluding present. Instantaneous causality is defined as:

$$
\begin{equation*}
Y+X \text { if; } \sigma^{2}(X \mid \bar{U}, \overline{\bar{Y}})<\sigma^{2}(X \mid \bar{U}) \tag{3.27}
\end{equation*}
$$

where two lines above $Y$ refer to present and past history values of $Y$. Equation (3.27) says we are better able to predict $X$ if the present value of $Y$ is included. The last type of causality is the causality lag. Its definition is the following:

$$
\begin{equation*}
Y \rightarrow X \text { if } ; \sigma^{2}(X \mid U-Y(k))<\sigma^{2}(X \mid U-Y(k+1)) \tag{3.28}
\end{equation*}
$$

at least lag $m$ of $Y$ is required for $Y$. Knowing the lagged $Y$ which is less than $m$ will be no help in improving the prediction of $X$.

When some of the information in set $U$ is irrelevant, we can ignore it and use only those relevant. For example, if the only relevant information are lagged values of $X$ and $Y$, then $Y$ is said to cause $X$ in Granger sense if:

$$
\begin{equation*}
\sigma^{2}(X \mid \bar{X}, \bar{Y})<\sigma^{2}(X \mid \bar{X}) \tag{3.29}
\end{equation*}
$$

However, spurious causality could arise if we exclude relevant information from the information set. For our purpose, we will use the linear causality in mean with respect to a specified information set (Sargent, 1979). The following are examples of linear causality:

$$
\begin{align*}
& x_{t}=\sum_{j=1}^{m} a_{j} X_{t-j}+\sum_{j=1}^{m} b_{j} Y_{t-j}+\varepsilon_{t} \\
& Y_{t}=\sum_{j=1}^{m} c_{j} X_{t-j}+\sum_{j=1}^{m} d_{j} Y_{t-j}+n_{t} \tag{3.30}
\end{align*}
$$

where $\left\{\varepsilon_{t}, \eta_{t}\right\}$ is the process of innovations

$$
\text { or } \left.\begin{array}{rl}
\varepsilon_{t} & =x_{t}-\hat{E}\left(x_{t} \mid \Omega_{t-1}\right) \text { and }  \tag{3.31}\\
n_{t} & =Y_{t}-\hat{E}\left(Y_{t} \mid \Omega_{t-1}\right)
\end{array}\right\}
$$

$$
\text { and, } \Omega_{t-1}=\left(Y_{t-1}, Y_{t-2}, \ldots X_{t-1}, X_{t-2} \ldots\right)
$$

$\hat{\mathrm{E}}$ is 1inear least-squares projection operator.
The following assumptions are needed:
i) $\left(Y_{t}, X_{t}\right)$ is a jointly covariance stationary time series.
ii) $E\left(\varepsilon_{t} X_{t-j}\right)=E\left(\varepsilon_{t} Y_{t-j}\right)=E\left(\eta_{t} X_{y-j}\right)=E\left(\eta_{t} Y_{t-j}\right)=0$ for all $j \geq 1$.

The orthogonal condition ii) implies

$$
E\left(\varepsilon_{t} \varepsilon_{t-\tau}\right)=E\left(\varepsilon_{t} \eta_{t-\tau}\right)=E\left(\eta_{t} \eta_{t-\tau}\right)=0
$$

for all nonzero of $\tau$.
If $Y$ fails to Granger-cause $X,{ }^{2}$ there exists a family of

[^0]\[

$$
\begin{equation*}
Y_{t}=\sum_{j=1}^{\infty} h_{j} x_{t-j}+u_{t} \tag{3.32}
\end{equation*}
$$

\]

expressing $Y_{t}$ as a one-sided distributed lag of $X$. This definition will be used in Chapter V.

CHAPTER 4. THE FORMULATION OF THE SOYBEAN MARKET

The model presented here is the decision-making behavior of producers and consumers in the soybean market. The decision rules - acreage planted, on-farm inventory, soybean crush and off-farm inventory -- are derived from a dynamic and stochastic framework where farmers and processors are assumed to maximize the expected present value of their income stream subject to dynamic and stochastic technology and their information. The decision rules are a function of, among other things, conditional expected future prices. Three price expectations are applied - rational, adaptive, and cash-futures expectations.

Under rational expectations, agents are assumed to know the actual distributions of exogenous and endogenous variables. Hence, the agents' decision rules depend upon the parameters underlying the structure of the soybean market and the parameters which characterize exogenous variables. Any changes in government policies affect decision rules such that the structural equations vary with such policies. Prediction of such policy changes requires identification of the structural model. The dynamic nature of the soybean market and uncontrollable shocks give rise to fluctuations of decision rules.

There are two sectors - producers and consumers of soybeans. Producers decide upon acreage planted to soybeans at time $t$ and decide upon the level of inventory to be carried over to the next period. Consumers buy soybeans for crushing into soybean meal and soybean oil. They also carry soybeans over for future usage and for speculation.

Soybean production is assumed to be a function of soybean acreage planted ( $a_{t}$ ), which is a decision variable. Farmers (producers) face production costs, which are the direct cost of producing soybeans, adjustment cost of acreage planting and storage cost (including opportunity cost) of holding soybeans. The adjustment cost of acreage planting represents additional changes in acreage planted from one crop year to the other. The adjustment cost is assumed to be a quadratic function, which is necessary for a dynamic model. Inventory cost includes the transaction cost of storing soybeans and the opportunity cost of lost revenue due to a possible change in the price of soybeans. For equilibrium, total demand for soybeans, i.e., soybeans bought by processors (crush), exports, change in inventory on-farm, and off-farm and change in soybean stock owned by the Commodity Credit Cooperation (CCC), must equal production plus beginning stock.

Soybean meal and oil production are assumed to equal the multiplication between soybean meal and oil yield and soybean crushed (sc ${ }_{t}$ ). Equilibrium is also imposed in this market. The cost of producing soybean meal and soybean oil is the cost of beans, adjustment cost of crushing and inventory cost. The linkage between soybeans and its products is through soybean price farmers receive ( $\mathrm{P}_{\mathrm{t}}$ ) and the wholesale price ( $\mathrm{PS} \mathrm{t}_{\mathrm{t}}$ ). Price, $P_{t}$, determines farmers' decision to plant soybeans, while PS $_{t}$ determines the demand for crush. The difference between $P_{t}$ and $P S_{t}$ reflects the margin between these two prices.

Model of Soybean Market
Soybean sector
The following equations belong to the $i^{\text {th }}$ farmer at time period $t$ where subscript " 1 " is omitted, and $i=1,2,3 \ldots N$. Definitions of all variables are reported in Appendix D. Capital letters represent aggregate notations, while small letters are for a representative agent. The time index " $t$ " in the following structural model refers to quarter. The first quarter of the soybean model in this study begins at planting time, in contrast to the traditional crop year where the first quarter begins at harvest time. The rearrangement is made to simplify the formation of the model. The time index " $t$ " takes values at $t=0,1,2$, ... unless stating otherwise.

Production of soybeans

$$
\begin{equation*}
s b(t+2)=y a(t) \tag{4.1}
\end{equation*}
$$

$a(t)$ takes positive values at $t=0,4,8, \ldots$ and zero otherwise. $s b$, thus, takes positive values at $t+2=2,6,10, \ldots$.

Soybean sold

$$
\begin{equation*}
s b s_{t}=s b b_{t}+s b x_{t} \tag{4.2}
\end{equation*}
$$

Soybean bought by processor

$$
\begin{equation*}
s b b_{t}=s c_{t}+\left(s b c_{t}-s b c_{t-1}\right) \tag{4.3}
\end{equation*}
$$

Total demand for soybeans

$$
\begin{array}{r}
\operatorname{sbd}_{t}=\operatorname{sbs}_{t}+\operatorname{sbhf}_{t}-\operatorname{sbhf}_{t-1} \\
+\operatorname{sbhc}_{t}-\operatorname{sbhc}_{t-1} \tag{4.4}
\end{array}
$$

- Soybean identity

$$
\begin{equation*}
s b_{t}+s b h f_{t-1}=s b s_{t}+s b h f_{t}+s b h c_{t}-s b h c_{t-1} \tag{4.5}
\end{equation*}
$$

Farm price 1inkage

$$
\begin{equation*}
P_{t}=\beta P S_{t}+s_{t} ; 0<\beta<1 \tag{4.6}
\end{equation*}
$$

Cost of soybean production

$$
\begin{equation*}
C 1_{t}=\alpha 1 a_{t} \quad ; t=0,4,8, \ldots T-4 \tag{4.7}
\end{equation*}
$$

where $\alpha 1$ represents average cost of production per acre.
Adjustment cost of acreage planted

$$
C 2_{t}=\frac{d 1}{2}\left(a_{t}-a_{t-4}\right)^{2} ; t=0,4,8, \ldots(4.8)
$$

Inventory holding cost of soybean on-farm

$$
\begin{equation*}
C 3_{t}=d 2 \operatorname{sbhf}_{t}+\frac{d 3}{2}\left(\operatorname{sbh} f_{t}\right)^{2}+\frac{d 4}{2}\left(\operatorname{sbhf}_{t}-\operatorname{sbhf} t-1\right)^{2} \tag{4.9}
\end{equation*}
$$

Cost of soybean bought

$$
\begin{equation*}
\mathrm{C4}{ }_{t}=\mathrm{PS}_{t} \mathrm{sbb}_{t} \tag{4.10}
\end{equation*}
$$

Adjustment cost of crushing

$$
\begin{equation*}
\mathrm{C} 5_{t}=\frac{g 0}{2} s c_{t}^{2}+\frac{g 1}{2}\left(s c_{t}-s c_{t-1}\right)^{2} \tag{4.11}
\end{equation*}
$$

Inventory cost of soybean off-farm

$$
\begin{equation*}
\mathrm{c} 6_{t}=\frac{\mathrm{g} 2}{2} s b c_{t}^{2}+\frac{g 3}{2}\left(s b c_{t}-s b c_{t-1}\right)^{2} \tag{4.12}
\end{equation*}
$$

Soybean meal and soybean oil sector
The following equations belong to processor " $j$ " where the subscript " j " is omitted, and $\mathrm{j}=1,2,3, \ldots$ K.

Soybean meal production

$$
\begin{equation*}
s_{t}=s o m s c * 50 * s_{t} \tag{4.13}
\end{equation*}
$$

Soybean oil production

$$
\begin{equation*}
\mathrm{so}_{\mathrm{t}}=\operatorname{soosc} * 100 * \mathrm{sc}_{\mathrm{t}} \tag{4.14}
\end{equation*}
$$

Soybean meal identity

$$
\begin{equation*}
s m_{t}+s m h_{t-1}=s m d m_{t}+s m x_{t}+s m h_{t} \equiv s m d_{t} \tag{4.15}
\end{equation*}
$$

Soybean oil Identity

$$
\begin{equation*}
s_{t}+\operatorname{soh}_{t-1}=\operatorname{sodm}_{t}+\operatorname{sox}_{t}+\operatorname{soh}_{t} \equiv \operatorname{sod}_{t} \tag{4.16}
\end{equation*}
$$

Total demand for soybean meal

$$
\begin{equation*}
\operatorname{smd}_{t}=m 0+m 1 \mathrm{PM}_{t}+\mathrm{m} 2 \mathrm{z1} 1_{t}+\ell_{1 t} \tag{4.17}
\end{equation*}
$$

Total demand for soybean of

$$
\begin{equation*}
\operatorname{sod}_{t}=k 0+k 1 P 0_{t}+k 22_{t}+\ell_{2 t} \tag{4.18}
\end{equation*}
$$

where $\mathrm{Z1} t_{t}$ and $\mathrm{Z} 2_{t}$ are other variables which represent demand shift. The time index " $t$ " in equations (4.4) through (4.18) take values at $t=0,1$, $2,3, \ldots$ for all variables except $s b_{t} \cdot \ell_{1 t}$ and $\ell_{2 t}$ are stochastic errors. The Rational Expectations Model of the Soybean Market

Given the structure of our market in equations (4.1) through (4.18), each producer (farmer) and consumer (processor) will maximize the expected value of the discounted present value of his (her) income stream.

A representative farmer will maximize:
${ }^{1}$ The figures 50 and 100 are required to adjust for units.

$$
\begin{align*}
V(0)= & E_{0} \sum_{t=0}^{\infty} b^{t}\left(P(t) \operatorname{sbs}(t)-\alpha 1 a(t)-\frac{d 1}{2}(a(t)-a(t-4))^{2}\right. \\
& -d 2 \operatorname{sbhf}(t)-\frac{\left.d 3 \operatorname{sbhf}(t)^{2}-\frac{d 4}{2}(\operatorname{sbhf}(t)-\operatorname{sbhf}(t-1))^{2}\right)}{} \tag{4.19}
\end{align*}
$$

where " $b$ " is a discount factor which is greater than zero and less than one. The index " t " takes values at $\mathrm{t}=0,1,2, \ldots$ for all variables except $a(t)$ which $t=0,4,8, \ldots$ Equation (4.19) is maximized over $\left\{a_{t}\right\}$ and $\left\{\operatorname{sbhf}_{t}\right\}$, given their initial values $a(t-j), \operatorname{sbhf}(t-j)$, and information accumulated up to time $t(\Omega(t-1))$, where

$$
\begin{aligned}
\Omega(t-1)= & \{a(t-1), a(t-2), \ldots \operatorname{sbhf}(t-1), \operatorname{sbhf}(t-2), \ldots \\
& P(t-1), P(t-2), \ldots \operatorname{PS}(t-1), \operatorname{PS}(t-2), \ldots \operatorname{sc}(t-1), \\
& \operatorname{sc}(t-2), \ldots \operatorname{sbc}(t-1), \operatorname{sbc}(t-2), \ldots \operatorname{PM}(t-1), \\
& P M(t-2), \ldots \operatorname{PO}(t-1), \operatorname{PO}(t-2), \ldots Z 1(t-1), Z 1(t-2), \\
& \ldots Z 2(t-1), Z 2(t-2), \ldots\}
\end{aligned}
$$

Using (4.1) and (4.2), we can write (4.19) as the following:

$$
\begin{align*}
V(0)= & E_{0} \sum_{t=0}^{\infty} b^{t}(P(t)(y a(t)+\operatorname{sbhf}(t-1)-\operatorname{sbhf}(t) \\
& -\operatorname{sbhc}(t)+\operatorname{sbhc}(t-1))-\alpha 1 a(t)-\frac{d 1}{2}(a(t)-a(t-4))^{2} \\
& \left.-d 2 \operatorname{sbhf}(t)-\frac{d 3}{2} \operatorname{sbhf}(t)^{2}-\frac{d 4}{2}(\operatorname{sbhf}(t)-\operatorname{sbhf}(t-1))^{2}\right) \tag{4.20}
\end{align*}
$$

Taking the derivative of (4.20) with respect to $\{a(t)\}$ where $t=0,4,8$, ... , we get:
$-b^{t} \alpha 1-d 1 b^{t}(a(t)-a(t-4))+b^{t+2} y P(t+2)+b^{t+4} d 1(a(t+4)-a(t))=0$
for $t=0,4,8, \ldots$

Rearranging (4.21), and using the lagged operator " L ", we get:

$$
\begin{array}{r}
\left(1-b^{-4}\left(1+b^{4}\right) L^{4}+b^{-4} L^{8}\right) a(t+4)=\left(b^{4} d 1\right)^{-1}\left(\alpha 1-b^{2} y P(t+2)\right) \\
\text { for } t=0,4,8, \ldots \tag{4.22}
\end{array}
$$

The transversarity condition of $a(t)$ is:

$$
\begin{equation*}
\lim _{T \rightarrow \infty} \cdot\left\{b^{T+4} d 1 \cdot(a(T+4)-a(T))-b^{T} d 1(a(T)-a(T-4))-b^{T} \alpha 1+b^{T+2} y P(T+2)\right\}=0 \tag{4.23}
\end{equation*}
$$

Equation (4.22) can be written as: ${ }^{2}$

$$
\begin{align*}
\left(1-L^{4}\right) a(t+4) & =\left(-b^{-4} L^{4}\right)^{-1}\left(1-b^{4} L^{-4}\right)^{-i}\left(b^{4} d 1\right)^{-1}\left(\alpha 1-b^{2} y P(t+2)\right) \\
& =\left(\frac{\left(-b^{4} L^{-4}\right)\left(b^{4} d 1\right)^{-1}}{\left(1-b^{4} L^{-4}\right)}\right)\left(\alpha 1-b^{2} y P(t+2)\right), \text { or } \\
a(t+4) & =a(t)-\frac{1}{d 1} \sum_{k=0}^{\infty}(b)^{4 k}\left(\alpha 1-b^{2} y P(t+6+4 k)\right. \\
& =a(t)-\left(\frac{\alpha 1}{d 1-(1-b)}\right)+\left(\frac{b^{2} y}{d 1}\right)_{k=0}^{\infty}(b)^{4 k} P(t+6+4 k) \\
& \text { for } t=0,4,8, \ldots \tag{4.24}
\end{align*}
$$

Equation (4.24) is not yet a decision rule for $a(t)$, because it is a function of future prices, which are unknown at the time making planting decision. To get the explicit decision rule, we need to write future

$$
\begin{aligned}
2\left(1-b^{-4}\left(1+b^{4}\right) L^{4}+b^{-4} L^{8}\right) & =\left(1-L^{4}\right)\left(1-b^{-4} L^{4}\right) \\
& =\left(1-L^{4}\right)\left(-b^{-4} L^{4}\right)\left(1-b^{4} L^{-4}\right)
\end{aligned}
$$

where $L^{-4} a(t)=a(t+4)$, and $L^{4} a(t)=a(t-4)$.
terms in an observable form. Now, taking the derivative of (4.20) with respect to $\{\operatorname{sbhf}\}_{t=0}^{\infty}$, we get the following system of Euler equations:

$$
\begin{gather*}
-b^{t} P(t)+b^{t+1} P(t+1)-b^{t} d 2-b^{t} d 3 \operatorname{sbh} f(t) \\
-b^{t} d 4(\operatorname{sbhf}(t)-\operatorname{sbhf}(t-1))+b^{t+1} d 4(\operatorname{sbh} f(t+1)-\operatorname{sbh}(t))=0 \\
\text { for } t=0,1,2,3, \ldots \tag{4.25}
\end{gather*}
$$

The transversarity condition for $\operatorname{sbh}(t)$ is:

$$
\begin{gather*}
\lim _{T \rightarrow \infty}-b^{T} P(T)+b^{T+1} P(T+1)-b^{T} d 2-b^{T} d 3 \operatorname{sbhf}(T) \\
-b^{T} d 4(\operatorname{sbh} f(T)-\operatorname{sbhf}(T-1))+b^{T+1} d 4(\operatorname{sbh} f(T+1)-\operatorname{sbh} f(T))=0 \tag{4.26}
\end{gather*}
$$

Rearranging (4.25), we get:

$$
\begin{align*}
\left(1-\left(\frac{d 3}{b d 4}+\right.\right. & \left.\left.b^{-1}(1+b)\right) L+b^{-1} L^{2}\right) \operatorname{sbhf}(t+1) \\
& =(b d 4)^{-1}(P(t)-b P(t+1)+d 2) \\
& =-(b d 4)^{-1}(b P(t+1)-P(t)-d 2)  \tag{4.27}\\
& \text { for } t=0,1,2,3, \ldots T-1
\end{align*}
$$

Let $\rho 1$ and $\rho 2$ be the two distinct roots of (4.27). We can write it as:

$$
(1-\rho 1 L)(1-\rho 2 L) \operatorname{sbhf}(t+1)=-(b d 4)^{-1}(b P(t+1)-P(t)-d 2) \quad(4.28)
$$

If $\rho 1$ is the smallest root, then we get $\rho 1<1<\frac{1}{b}<\rho 2$. Also, the process $P(t)$ is an exponential order less than $1 / b$ (this concept will be used throughout this chapter). Let $L^{-1} \operatorname{sbhf}(t)=s b h f(t+1)$, then equation (4.28) can be written as:

$$
(1-\rho 1 L) \operatorname{sbhf}(t+1)=\frac{(b d 4 \rho 2 L)^{-1}(b P(t+1)-P(t)-d 2)}{\left(1-(\rho l b) L^{-1}\right)} \text {, or }
$$

$$
\left.\begin{array}{rl}
\operatorname{sbhf}(t+1) & =\rho \operatorname{sbhf}(t)+\left(\frac{\rho 1}{d 4}\right)_{i=0}^{\infty}\left(\frac{1}{\rho 2}\right)^{i} \begin{array}{l}
(b P(t+i+2)-P(t+1)-d 2) \\
\text { for } t=0,1,2, \ldots(4.29)^{3}
\end{array} \\
\text { where } \rho 1+\rho 2=\frac{d 3}{b d 4}+b^{-1}(1+b)>0 \\
\rho 1 \rho 2 & =1 / b \tag{4.30}
\end{array}\right\} \quad \text {, }
$$

Equation (4.29) is not yet a decision rule because of the future price $P(t+i)$. Before solving for explicit function of $a(t+1)$ and $\operatorname{sbh} f(t+1)$, let us turn to soybean processors.

A representative soybean processor maximizes the following expression:

$$
\begin{align*}
J(0)=E_{0} \sum_{t=0}^{\infty} & \delta^{t}\{P M(t) \operatorname{smd}(t)+P O(t) \operatorname{sod}(t)-P S(t) \operatorname{sbb}(t) \\
& -\frac{g 0 \operatorname{sc}(t)^{2}-\frac{g l}{2}(\operatorname{sc}(t)-\operatorname{sc}(t-1))^{2}}{} \\
& \left.\left.-\frac{g 2}{2} \operatorname{sbc}(t)\right)^{2}-\frac{g 3}{2}(\operatorname{sbc}(t)-\operatorname{sbc}(t-1))^{2}\right\} \tag{4.31}
\end{align*}
$$

over the processes $s c(t)$ and $s b c(t)$, given all initial information at period $t(\Omega(t-1))$. Equation (4.31) holds for $t=0,1,2,3, \ldots T-1$. Imposing condition (4.15), (4.3), (4.16), and (4.14), we can write (4.31) as:
$3 \frac{1}{(1-\rho 2 L)}=-\frac{(\rho 2 L)^{-1}}{(1-(\rho 2 L))^{-1}}=-(\rho 1 b) \frac{L^{-1}}{(1-(\rho 2 L))^{-1}}$.

$$
\begin{aligned}
& J(0)=E_{0} \sum_{t=0}^{\infty} \delta^{t} / \mathrm{PM}(t)\left(\operatorname{somsc}^{*} \operatorname{sc}(t)+\operatorname{smh}(t-1)\right)+\mathrm{PO}(t)\left(\mathrm{soosc}^{*}\right. \\
& s c(t)+\operatorname{soh}(t-1))-\operatorname{PS}(t)(\operatorname{sc}(t)+ \\
& \operatorname{sbc}(t)-\operatorname{sbc}(t-1))-\frac{g 0 \operatorname{sc}(t)^{2}}{2} \\
& -\frac{g 1}{2}(\operatorname{sc}(t)-\operatorname{sc}(t-1))^{2} \\
& \left.-\frac{\mathrm{g} 2}{2} \operatorname{sbc}(\mathrm{t})^{2}-\frac{\mathrm{g} 3}{2}(\mathrm{sbc}(\mathrm{t})-\mathrm{sbc}(\mathrm{t}-1))^{2}\right) \quad(4.32)^{4}
\end{aligned}
$$

Taking the derivative of equation (4.32) with respect to $\{s c\}_{t=0}^{\infty}$, we get the Euler system

$$
\begin{align*}
& \delta^{t}\left\{\operatorname{PM}(t) *_{\text {somsc }}+P O(t) *_{\text {soosc }}-P S(t)\right\}-\delta^{t} g 0 \operatorname{sc}(t) \\
- & \delta^{t} g 1(\operatorname{sc}(t)-s c(t-1))+\delta^{t+1} g 1(s c(t+1)-s c(t))=0 \tag{4.33}
\end{align*}
$$

The transversality condition is: .

$$
\begin{align*}
& \lim _{\mathrm{T} \rightarrow \infty}\left\{\delta^{\mathrm{T}}(\mathrm{PM}(\mathrm{~T}) * \text { somsc }+\mathrm{PO}(\mathrm{~T}) * \text { soosc }-\mathrm{PS}(\mathrm{~T}))\right. \\
& -\delta^{\mathrm{T}} \mathrm{~g} 0 \operatorname{sc}(\mathrm{~T})-\delta^{\mathrm{T}} \mathrm{~g} 1(\mathrm{sc}(\mathrm{~T})-\mathrm{sc}(\mathrm{~T}-1)) \\
& \left.\quad+\delta^{\mathrm{T}+1} \mathrm{~g} 1(\mathrm{sc}(\mathrm{~T}+1)-\mathrm{sc}(\mathrm{~T}))\right\}=0 \tag{4.34}
\end{align*}
$$

Making use of the lagged operator "L" and rearranging terms, we can rewrite equation (4.33) as the following

$$
\left(1-(1+1 / 6+g 0 / \delta g 1) L+1 / \delta L^{2}\right) \operatorname{sc}(t+1)=-(1 / \delta g 1) \operatorname{sBCM}(t)
$$

[^1]where $\operatorname{SBCM}(t)=\frac{P M(t)}{20} *_{\text {somsc }}+P O(t) *$ soosc $-P S(t)$. Using the same technique as in (4.28), we can write (4.35) as:
\[

$$
\begin{equation*}
(1-\lambda 1 L)(1-\lambda 2 L) \operatorname{sc}(t+1)=-(\delta g 1)^{-1} \operatorname{sBCM}(t) \tag{4.36}
\end{equation*}
$$

\]

where $\lambda 1$ and $\lambda 2$ are two distinct roots, and $\lambda 1$ is the smallest root. Equation (4.36) has the following solution:

$$
\begin{gathered}
\operatorname{sc}(t+1)=\lambda 1 \operatorname{sc}(t)+\frac{\lambda 1}{g 1} \sum_{i=0}^{\infty}(1 / \lambda 2)^{i}(\operatorname{SBCM}(t+i+1)) \quad(4.37) \\
\text { for } t=0,1,2, \ldots
\end{gathered}
$$

$$
\text { where } \left.\begin{array}{rl}
\lambda 1+\lambda 2 & =(1+1 / \delta+g 0 / \delta g 1)  \tag{4.38}\\
\lambda 1 \lambda 2 & =1 / \delta
\end{array}\right\}
$$

Taking the derivative of (4.31) with respect to $\{\operatorname{sbc}\}_{\mathrm{t}=0}^{\infty}$, we get

$$
\begin{align*}
& \delta^{t+1} g 3 \cdot(\operatorname{sbc}(t+1)-\operatorname{sbc}(t))-\delta^{t} g 3(\operatorname{sbc}(t)-\operatorname{sbc}(t-1)) \\
& \delta^{t} g 2 \operatorname{sbc}(t)+\delta^{t+1} \operatorname{PS}(t+1)-\delta^{t} \operatorname{PS}(t)=0 \tag{4.39}
\end{align*}
$$

and the transversality is:

$$
\begin{gather*}
\lim _{\mathrm{T} \rightarrow \infty}\left\{\delta^{\mathrm{T}+1} \mathrm{~g} 3(\mathrm{sbc}(\mathrm{~T}+1)-\operatorname{sbc}(\mathrm{T}))-\delta^{T} \mathrm{~g} 3(\operatorname{sbc}(\mathrm{~T})-\operatorname{sbc}(\mathrm{T}-1))\right. \\
 \tag{4.40}\\
\left.\delta^{T} \mathrm{~g} 2 \mathrm{sbc}(\mathrm{~T})+\delta^{T+1} \mathrm{PS}(\mathrm{~T}+1)-\delta^{T} \mathrm{PS}(\mathrm{~T})\right\}=0
\end{gather*}
$$

As before, the solution which satisfies the system of Euler equation is:

$$
\operatorname{sbc}(t+1)=\theta 1 \operatorname{sbc}(t)+(\theta 1 / \mathrm{g} 3) \sum_{i=0}^{\infty}(1 / \theta 2)^{i}(\delta \operatorname{PS}(t+1+2
$$

$$
-\operatorname{PS}(t+i+1)), \text { for } t=0,1,2,(4.41)
$$

where $\left.\begin{array}{rl}\theta 1+\theta 2 & =1+\frac{1}{\delta}+(\mathrm{g} 2 / \mathrm{g} 3 \delta) \\ \theta 1 \theta 2 & =1 / \delta\end{array}\right\}$.

Aggregating the Euler equations across individual farmers and processors, we get U.S. soybean acreage planted ( $A_{t}$ ), U.S. soybean inventory on-farm ( SBHF $_{t}$ ), U.S. soybean crush ( SC $_{t}$ ) and U.S. soybean commercial inventory off-farm ( SBC $_{t}$ ). By aggregating we imply that the agents' behavior in this market are the same, and their perceptions on the aggregate law of motion for the exogenous variables are correct. 5

For the market to be clear at harvest time ( $t+2$ ), total demand for soybeans must equal total supply of soybeans at a market price at harvest time ( $P S_{t+2}$ ). The market clearing condition is:

$$
\begin{aligned}
\mathrm{SC}_{\mathrm{t}+2}+\mathrm{SBX}_{\mathrm{t}+2}+\mathrm{SBC}_{\mathrm{t}+2}+\mathrm{SBHF}_{\mathrm{t}+2}+\mathrm{SBHC}_{\mathrm{t}+2}= & \mathrm{SB}_{\mathrm{t}+2}+\mathrm{SBC}_{\mathrm{t}+1} \\
& +\mathrm{SBHF}_{\mathrm{t}+1} \\
& + \text { SBHC }_{\mathrm{t}+1}
\end{aligned}
$$

To get the market price $\mathrm{PS}_{\mathrm{t}+2}$ which satisfies (4.43), we substitute the aggregate Euler equations into (4.43). The market price $\mathrm{PS}_{\mathrm{t}+2}$ is:
$5_{\text {There }}$ is no reason to believe that the equilibrium imposed on this derivation is efficient. The equilibria depend very much on the expectations of the agents, and these expectations may be radically different. If agents have rational expectations, then the economy will be efficient.

$$
\begin{align*}
& P S_{t+2}=\left(\frac{g 3}{\delta g 1}+\frac{1}{\delta}+1\right) P S_{t+1}-\frac{1}{\delta}{ }^{\prime} P S_{t}-g 3 y b^{-4}\left(1+b^{4}\right) A_{t-4} \\
& +g 3 y b^{-4} A_{t-8}-\frac{g 3 y}{b^{4} d 1} \alpha 1+\frac{g 3 y^{2}}{b^{2} d 1} P_{t-2}-\frac{g 3}{b d 4} P_{t}+\frac{g 3}{d 4} P_{t+1} \\
& -\frac{g 3}{d 4} P_{t+2}+g 3\left(1+\frac{1}{\delta}+\frac{g 0}{\delta g 1}\right) S C_{t+1}-\frac{g 3}{\delta} S C_{t} \\
& -\mathrm{g} 3 \frac{\text { somsc }}{20 \delta \mathrm{~g} 1}{ }^{\mathrm{PM}}{ }_{\mathrm{t}+1}-\mathrm{g} 3 \frac{\text { soosc }}{\delta \mathrm{g} 1} \mathrm{PO}_{\mathrm{t}+1}+\mathrm{g} 3 \mathrm{SBX} \mathrm{t}+2 \\
& +g 3\left(1+\frac{1}{\delta}+\frac{g 2}{\delta g^{3}}\right) S B C_{t+1}-g 3\left(1+\frac{2}{\delta}+\frac{g 2}{\delta g^{3}}\right) S B C_{t} \\
& \left.+\frac{g 3}{\delta} S B C C_{t-1}+g 3 \frac{(d 3}{b d 4}+b^{-1}(1+b)\right) \text { SBHF }_{t+1} \\
& -g 3\left(\frac{d 3}{b d 4}+\frac{2}{b}+1\right) \text { SBHF }_{t}+\frac{g 3}{b} \text { SBHF }_{t-1} \\
& +g 3 \text { SBHC }_{t+2}-\mathrm{g} 3 \text { SBHC }_{t+1} \tag{4.44}
\end{align*}
$$

The market clearing prices for soybean meal and soybean oil are:

$$
\begin{align*}
& P M_{t+2}=-\frac{\text { somsc }^{2} * 2.5}{\mathrm{ml} \mathrm{\delta} \mathrm{gl}} \mathrm{PM}_{\mathrm{t}+1}-\frac{\text { somsc*soosc*50 }^{\mathrm{ml} \delta \mathrm{~g} 1}}{\mathrm{PO}}{ }_{\mathrm{t}+1} \\
& +\frac{\text { somsc*50 }}{\mathrm{mldg} 1} \mathrm{PS}_{\mathrm{t}+1}+\frac{\text { somsc*50 }}{\mathrm{ml}}\left(1+\frac{1}{\delta}+\frac{\mathrm{g} 0}{\delta \mathrm{gl}}\right) \mathrm{SC}_{\mathrm{t}+1} \\
& -\frac{1}{\mathrm{ml}}{ }^{\ell}{ }_{1 \mathrm{t}}-\frac{\mathrm{somsc} * 50}{\mathrm{ml} \delta} \mathrm{SC}_{\mathrm{t}}-\frac{\mathrm{m} 2}{\mathrm{ml}} \mathrm{Z1}_{\mathrm{t}+2}-\frac{\mathrm{m} 0}{\mathrm{ml}}+\frac{1}{\mathrm{~m} 1} \mathrm{SMH}_{\mathrm{t}+1}  \tag{4.45}\\
& \mathrm{PO}_{\mathrm{t}+2}=\frac{\text { Soosc }^{*} 100}{\mathrm{k} 1}\left(1+\frac{1}{\delta}+\frac{\mathrm{g} 0}{\delta \mathrm{~g} 1}\right) \mathrm{SC} \mathrm{t}_{\mathrm{t}+1}-\frac{1}{\delta \mathrm{kI}} \mathrm{SC} \mathrm{~S}_{\mathrm{t}} \\
& -\frac{\text { somsc }}{20 \delta g 1 \mathrm{kl}} \mathrm{PM}_{\mathrm{t}+1}-\frac{\text { soosc }}{\delta \mathrm{gl} \mathrm{kl}} \mathrm{PO}_{\mathrm{t}+1}+\frac{1}{\delta \mathrm{gl} \mathrm{kl}} \mathrm{PS}_{\mathrm{t}+1} \\
& +\frac{1}{\mathrm{k} 1} \mathrm{SOH}_{\mathrm{t}+1}-\frac{\mathrm{k} 2}{\mathrm{k} 1} \mathrm{Z2}_{\mathrm{t}+2}-\frac{\mathrm{k} 0}{\mathrm{kI}}-\frac{1}{\mathrm{kI}} \ell_{2 \mathrm{t}} \tag{4.46}
\end{align*}
$$

Thus, in summary, the equations for the four choice variables are as following:

$$
\begin{align*}
& A_{t+4}=A_{t}-\frac{1}{d} \sum_{k=0}^{\infty}(b)^{4 k}\left(a 1-b^{2} y E_{t}\left(P_{t+6+4 k}\right)\right)  \tag{4.47}\\
& \text { for } t=0,4,8, \ldots \\
& \operatorname{SBHF}_{\mathrm{t}+2}=\rho 1 \mathrm{SBHF}_{\mathrm{t}+1}+\frac{\rho 1}{d 4} \sum_{i=0}^{\infty}(1 / \rho 2)^{i}\left(\mathrm{~b} \mathrm{E}_{\mathrm{t}}\left(\mathrm{P}_{\mathrm{t}+\mathrm{i}+3}\right)\right. \\
& \left.-E_{t}\left(P_{t+i+2}\right)-d 2\right)  \tag{4.48}\\
& \text { for } t=0,1,2, \ldots
\end{align*}
$$

where

$$
\begin{aligned}
\rho 1+\rho 2 & =\frac{d 3}{b d 4}+b^{-1}(1+b) \\
\rho 1 \rho 2 & =1 / b
\end{aligned}
$$

and

$$
\begin{gather*}
P_{t}=\beta P S_{t}+s_{t} \\
S C_{t+2}=\lambda 1 S C_{t+1}+\frac{\lambda I}{g 1} \sum_{i=0}^{\infty} \begin{array}{l}
(1 / \lambda 2)^{i} E_{t}\left(S B C M_{t+i+2}\right) . \\
\text { for } t=0,1,2,3, \ldots
\end{array} \tag{4.49}
\end{gather*}
$$

where $\lambda 1+\lambda 2=1+1 / \delta+g 0 / \delta g 1$

$$
\begin{align*}
& \lambda 1 \lambda 2=1 / \delta \\
& \mathrm{SBCM}_{\mathrm{t}+2}={\underset{\mathrm{PM}}{\mathrm{~L}+2}}{ }^{*} \text { somsc }+\mathrm{PO}_{\mathrm{t}+2}{ }^{*} \text { soosc }-\mathrm{PS} \mathrm{~S}_{\mathrm{t}+2} \\
& \mathrm{SBC}_{\mathrm{t}+2}=\theta 1 \mathrm{SBC}_{\mathrm{t}+1} \frac{\theta 1}{\mathrm{~g} 3} \sum_{\mathrm{I}=0}^{\infty}(1 / \theta 2)^{i}\left(\delta \mathrm{E}_{\mathrm{t}}\left(\mathrm{PS}_{\mathrm{t}+\mathrm{i}+3}\right)\right. \\
& \left.-E_{t}\left(P S_{t+i+2}\right)\right)  \tag{4.50}\\
& \text { for } t=0,1,2,3, \ldots
\end{align*}
$$

where

$$
\begin{aligned}
\theta 1+\theta 2 & =1+1 / \delta+g 2 / \delta g 3 \\
\theta 1 \theta 2 & =1 / \delta
\end{aligned}
$$

The expected value $E_{t}($.$) in equations (4.47) through (4.50) indicates the$ conditional expectation $E\left(. \mid \Omega_{t-1}\right)$, where $\Omega_{t-1}$ contains at least all past values of all endogenous and exogenous variables in the model
accumulated up to time $t$. Equation (4.47) says that the contingency plan for acreage planted is a function of soybean acreage planted last season, anticipated price received at harvest and for future time periods.

The acreage planted varies directly with the anticipated prices -or $\frac{\partial A_{t}}{\partial P_{t+j}}>0\left(\frac{\partial A_{t}}{\partial P S}{ }_{t+j}>0\right.$ as well $)$. The contingency plan for holding soybeans on-farm $\left(\right.$ SBHF $\left._{t}\right)$ depends upon its lag value and current and anticipated price changes. Farmers tend to hold more beans, either physical inventory or futures contracts, if they expect higher prices in the future. Soybean crush $\left(S C_{t}\right)$ at time $t$ depends upon beans crushed at t-1 and the whole anticipated future course of the soybean crushing margin (SBCM). A high price of beans at $t$ will.lower the quantity of beans crushed at $t$ or $\frac{\partial S C_{t}}{\partial P S_{t}}<0$.

The sign of $\partial S C_{t}$ and $\partial S C_{t}$ are expected to be positive. Soybean $\frac{\partial \overline{P M}_{t}}{} \quad \frac{\partial C_{t}}{\partial O_{t}}$ inventory off-farm is a function of its lag and the whole course of anticipated price changes in the future. We expect $\frac{\partial S B C_{t}}{\partial P S_{t+1}}$ to be positive and $\frac{\partial S B C_{t}}{\partial P S_{t}}$ to be nagative. Note that all coefficients in the four equations depend upon structural parameters underlying the model. In order to illustrate how the agents reacts to changes in price expectations, consider an example illustrated in Figure 4.1.

Panel (1) is the price linkage (equation (4.6)) between wholesale price $P S_{t}$ and price farmers receive $P_{t}$. Panel (2) and (3) are the demand for soybean crush $S C_{t}$ and exports $S B X_{t}$.


Figure 4.1. Graphical relationships of soybean complex

Panels (4) and (5) are soybean inventories on-farm SBHF $_{t}$ and soybean inventories off-farm $S B C_{t}$. In order to derive the seasonal supply curve Sl $t_{t}$, consider the following market condition:

$$
S C_{t}+S B X_{t}+S B H F_{t}+S B C_{t}+S B H C_{t}=S B_{t}+S B H F_{t-1}+S B C_{t-1}+S B H C_{t-1}
$$

This equation says that total demand -- soybean crush plus exports plus total inventories at the end of period $\left(\right.$ SBHF $_{t}+S B C_{t}+$ SBHC $_{t}$ ), equals to total supply -- production plus beginning stock. The demands for holding stocks at the end of quarter $t$ becomes the available supply at $t+1$. Thus, if holding demand is subtracted from the right-hand side, we get seasonal available supply such as $S l_{t}$ in Panel (6). The curve $S l_{t}$ is upward-sloping, because $S B H F_{t}$ and $S B C_{t}$ slope downward. At equilibrium we, therefore, obtain:

$$
\begin{align*}
S C_{t}+S B X_{t}= & \left(S B_{t}+S B H F_{t-1}+S B C_{t-1}+S B H C_{t-1}\right) \\
& -\left({S B H F_{t}}+S B C_{t}+S B H C_{t}\right) \tag{4.51}
\end{align*}
$$

The curve $D_{t}$ in Pane1 (6) is the sum of $S C_{t}$ and $S B X_{t}$. Let $S 0$ be the reference line referring to $\left(S B_{t}+S B H F_{t-1}+S B C_{t-1}+S B H C_{t-1}\right)$ at harvest . time (first quarter of the crop year), then equation (4.51) at harvest time can be written as:

$$
\begin{aligned}
& D_{t}=S 0-\left(S B H F_{t}+S B C_{t}+S B H C_{t}\right), \text { or } \\
& D_{t}=S 1_{t}, \text { and }
\end{aligned}
$$

equilibrium price $\mathrm{PS}_{t}$ is determined at $\mathrm{PS}_{0}$. Given $\mathrm{SMD}_{t}$ (total demand for soybean meal) and $S O D_{t}$ (total demand for soybean oil), we get the equilibrium price of soybean meal and oil at $\mathrm{PM}_{0}$ and $\mathrm{PO}_{0}$.

Now assume that agents expect the price of soybeans to increase in the next period. Given everything else unchanged, $S C_{t}$ and $S B X_{t}$ will
shift up to the right and thus $D_{t} \frac{\partial S C_{t}}{\partial P S_{t+j}}>0 . \quad S B H F_{t}$ and $S B C_{t}$ also shift to the right which makes $S 1_{t}$ shift to $S 2_{t}$. The new equilibrium price is higher at PS $_{1}$, and the price farmers receive is also higher at $P_{1}$. The quantity of soybean crush decreases. We assume for now that $S M D_{t}$ and $S O D_{t}$ stay unchanged; therefore, the new equilibrium price of soybean meal and oil increase at $\mathrm{PM}_{1}$ and $\mathrm{PO}_{1}$.

The non-choice variables $\mathrm{PS}_{t}, \mathrm{PM}_{t}, \mathrm{PO}_{t}$ are endogenous and have to be solved simultaneously with the four choice variables - $A_{t}, \operatorname{SBHF}_{t}, S_{t}$, and $S B C_{t}$. Thus, price expectations concerning $P S_{t}, \mathrm{PM}_{t}$ and $\mathrm{PO}_{\mathrm{t}}$ are endogenous. The endogenous price expectations are addressed by Muth (1961). The easier approach is to take prices as exogenous -- determine outside the model. It is worth mentioning that an attempt was made to treat $\mathrm{PM}_{t}$ and $\mathrm{PO}_{t}$ as exogenous. This attempt failed resulting from the Granger causality test.

The Decision Rules under Rational Expectations Hypothesis
It is known from the price linkage (4.6) that

$$
\begin{gathered}
P_{t}=\beta P S_{t}+s_{t} \text {, thus, } \\
E_{t}\left(P_{t+k}\right)=\beta E_{t}\left(P S_{t+k}\right), \text { where, } \\
E_{t}\left(s_{t+k}\right)=0 \text { for } k \geq 1
\end{gathered}
$$

Substituting $E_{t}\left(P_{t+k}\right)$ into equation (4.47) and (4.48), we get:

$$
\begin{equation*}
\left(A_{t+4}-A_{t}\right)=-\left(\frac{a 1}{d 1\left(1-b^{4}\right)}\right)+\left(\frac{b^{2} y}{d 1}\right) B \sum_{k=0}^{\infty}(b)^{4 k} E_{t}\left(P S_{t+6+4 k}\right) \tag{4.52}
\end{equation*}
$$

$$
\begin{align*}
\text { SBHF }_{t+2}= & \rho 1 S^{S B H F_{t+1}}+\frac{\rho b}{d 4} \beta \sum_{i=0}^{\infty}(1 / \rho 2) \dot{I}_{t}\left(P S_{t+i+3}\right) \\
& -\left(\frac{\rho 1}{d 4}\right) \sum_{i=0}^{\infty}(1 / \rho 2)^{i} E_{t}\left(P S_{t+i+2}\right) \\
& -\left(\frac{\rho 1}{d 4}\right) \frac{d 2}{(1-\rho l b)} \tag{4.53}
\end{align*}
$$

The other two choice variables are:

$$
\begin{align*}
& S C_{t+2}=\lambda 1 S C_{t+1}+\frac{\lambda 1}{g 1} \frac{\text { somsc }}{20} \sum_{i=0}^{\infty}(1 / \lambda 2)^{i} E_{t}\left(\mathrm{PM}_{t+1+2}\right) \\
& +\frac{\lambda I}{g I} \operatorname{soosc} \sum_{i=0}^{\infty}(1 / \lambda 2)^{i} E_{t}\left(P O_{t+i+2}\right) \\
& -\frac{\lambda 1}{g 1} \sum_{i=0}^{\infty}(1 / \lambda 2) E_{t}\left(P S_{t+i+2}\right)  \tag{4.54}\\
& S B C_{t+2} . .=\theta 1 S B C_{t+1}+\frac{\theta 1}{g^{3}} \sum_{i=0}^{\infty}(1 / \theta 2)^{i} E_{t}\left(P S_{t+i+3}\right) \\
& -\left(\frac{\theta 1}{g^{3}}\right)_{i=0}^{\infty}(1 / \theta 2)^{i} E_{t}\left(\text { PS }_{t+i+2}\right) \tag{4.55}
\end{align*}
$$

The rest of the restrictions are:

$$
\left.\begin{array}{rl}
\rho 1+\rho 2 & =\frac{d 3}{b d 4}+b^{-1}(1+b)  \tag{4.56}\\
\rho 1 \rho 2 & =1 / b \\
\lambda 1+\lambda 2 & =\frac{g 0}{\delta g 1}+1+\frac{1}{\delta} \\
\lambda 1 \lambda 2 & =1 / \delta \\
\theta 1+\theta 2 & =\frac{g 2}{\delta g}+1+1 / \delta \\
\theta 1 \theta 2=1 / \delta
\end{array}\right\}
$$

In order to write equations (4.52) through (4.55) into explicit forms, we use a time series interpretation on all variables in the model. ${ }^{6}$ We have shown from equation (4.44), (4.45) and (4.46) that $\mathrm{PS}_{t}, \mathrm{PM}_{t}$ and $P O_{t}$ are functions of its lagged values, lagged choice variables and other exogenous variables. In order to show how to find solutions to $E_{t}\left(P S_{t+k}\right)$, $E_{t}\left(P M_{t+k}\right)$ and $E_{t}\left(P O_{t+k}\right)$, we have to make assumptions concerning the variables on the right-hand side of the three price equations. Let $X_{1 t}$, $X_{2 t}$ and $X_{3 t}$ be covariance stationary time series vectors as the following:

$$
\begin{align*}
& x_{1 t}=B_{1} x_{1 t-1}+V_{1 t}  \tag{4.57}\\
& x_{2 t}=B_{2} x_{2 t-1}+v_{2 t}  \tag{4.58}\\
& x_{3 t}=B_{3} x_{3 t-1}+v_{3 t} \tag{4.59}
\end{align*}
$$

where $X_{1 t}, X_{2 t}$ and $X_{3 t}$ are vectors of current and lagged values of all variables in the price equations. If a variable in $X_{i t}$ has $r_{i}^{\text {th }}$ order of autoregressive realization, then we need $\left(r_{i}-1\right)$ lagged values in the $X_{i t}$ matrix. For example, taking only $\mathrm{PS}_{t}$ in $\mathrm{X}_{1 t}$, we have:
${ }^{6}$ If $P S_{t}$ is a covariance stationary time series, then $E_{t}\left(P_{t}\right)$ is a constant for all t. This implies that the infinite sum on the right-hand side of equations (4.52) through (4.55) converse to equilibrium level.

$$
\mathrm{X}_{1 \mathrm{t}}=\left[\begin{array}{l}
\mathrm{PS}_{\mathrm{t}} \\
\mathrm{PS}_{\mathrm{t}-1} \\
\mathrm{PS}_{\mathrm{t}-2} \\
\mathrm{PS}_{\mathrm{t}}-\mathrm{r}_{1}+1 \\
\cdot \\
\cdot \\
\cdot
\end{array}\right]=\left[\begin{array}{cccccc}
\gamma_{11} & \gamma_{12} & \cdots \gamma_{1 r_{1}} & & & \\
1 & 0 & 0 & 0 & & 0 \\
0 & 1 & 0 & 0 & & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
& \cdot & \cdot & \cdot & \cdot & \cdot
\end{array}\right]\left[\begin{array}{l}
\mathrm{PS}_{\mathrm{t}-1} \\
\mathrm{PS}_{\mathrm{t}-2} \\
\mathrm{PS}_{\mathrm{t}-\mathrm{r}_{1+2}} \\
\mathrm{PS}_{\mathrm{t}-\mathrm{r}_{1+1}} \\
\cdot \\
\cdot \\
.
\end{array}\right]+\mathrm{V}_{1 \mathrm{t}}
$$

where $\gamma_{11}, \gamma_{12}, \ldots \gamma_{1 r_{1}}$ are coefficients of $P_{t-1}, P S_{t-2}, \ldots S_{t-r_{1}}$.
Some of the $\gamma_{i j}$ may be zero, but $\gamma_{1 r_{1}}$ cannot be zero. We arrange $X_{2 t}$ and $X_{3 t}$ in the same fashion with $P M_{t}$ and $\mathrm{PO}_{t}$ being the first elements in those matrices.

Let $U_{1}, U_{2}$ and $U_{3}$ be row vectors with one being the first element and zero otherwise or

$$
U_{1}=(1,0,0 \ldots 0)
$$

Therefore, $\left.\underset{t}{E(P S}{ }_{t+k}\right)=U_{1} \underset{t}{E} X_{1 t+k}$ and

$$
\begin{aligned}
\sum_{k=0}^{\infty}(b)^{k} \underset{E}{E}\left(P S_{t+k}\right) & =U_{1} Q_{1} \sum_{k=0}^{\infty}\left(b \Lambda_{1}\right)^{k} Q_{1}^{-1} X_{t} \\
& =U_{1} Q_{1}\left[I-b \Lambda_{1}\right]^{-1} Q_{1}^{-1} X_{t}
\end{aligned}
$$

where $Q_{1}$ are eigenvectors of $B_{1}$ and

$$
\begin{aligned}
& \Lambda_{1} \text { are eigenvalues of } B_{1} \text { and } \\
& B=Q_{1} \Lambda_{1} Q_{1}^{-1}
\end{aligned}
$$

We assume that matrices $B_{1}, B_{2}$ and $B_{3}$ are known; therefore, we can rewrite equations (4.52) through (4.55) as:

$$
\begin{align*}
&\left(A_{t+4}-A_{t}\right)=-\left(\frac{1}{d_{1}}\right) \frac{\alpha_{1}}{i-B^{4}}+\left(\frac{b^{2} y}{d_{1}}\right) B U_{1} Q_{1}\left[I-b \Lambda_{1}\right]^{-1} Q_{1}^{-1} X_{1 t+4}(4 \\
& S B H F_{t+2}= \rho_{1} S B H F_{t+1}+\left(\frac{\rho_{1} b}{d_{4}}\right) B U_{1} Q_{1}\left[I-b \Lambda_{1}\right]^{-1} Q_{1}^{-1} X_{1 t+2} \\
&-\left(\frac{\rho_{1}}{d_{4}}\right) B U_{1} Q_{1}\left[I-b \Lambda_{1}\right]^{-1} Q_{1}^{-1} X_{1 t+1} \\
&-\left(\frac{\rho_{1}}{d_{4}}\right)\left(\frac{d_{2}}{1-\rho_{1} b}\right)  \tag{4.61}\\
& S C_{t+2}=\lambda_{1} S C_{t+1}+\left(\frac{\lambda_{1}}{g_{1}}\right) \frac{\operatorname{somsc}}{20} U_{2} Q_{2}\left[I-\lambda_{1} \delta \Lambda_{2}\right]^{-1} Q_{2}^{-1} X_{2 t+2} \\
&-\quad\left(\frac{\lambda_{1}}{g_{1}}\right) \operatorname{soosc} U_{3} Q_{3}\left[I-\lambda_{1} \delta \Lambda_{3}\right]^{-1} Q_{3}^{-1} X_{3 t+2} \\
&-\left(\frac{\lambda_{1}}{g_{1}}\right) U_{1} Q_{1}\left[I-\lambda_{1} \delta \Lambda_{1}\right]^{-1} Q_{1}^{-1} X_{1 t+2} \tag{4.62}
\end{align*}
$$

where $Q_{2}$ and $Q_{3}$ are eigenvectors and $\Lambda_{2}$ and $\Lambda_{3}$ are eigenvalues of $B_{2}$ and $B_{3}$, respectively.

$$
\begin{align*}
S B C_{t+2}= & \theta_{1} S B C_{t+1}+\left(\frac{\theta_{1}}{g_{3}}\right) \delta U_{1} Q_{1}\left[I-\theta_{1} \delta \Lambda_{1}\right]^{-1} Q_{1}^{-1} X_{1 t+2} \\
& -\left(\frac{\theta_{1}}{g_{3}}\right) U_{1} Q_{1}\left[I-\theta_{1} \delta \Lambda_{1}\right]^{-1} Q_{1}^{-1} X_{1 t+1} \tag{4.63}
\end{align*}
$$

The equations (4.44), (4.45), 4.46), (4.60), (4.61), 4.62), (4.63), structural equations, the restrictions (4.56), and the stochastic processes of all exogenous variables will be used in the estimation.

The rest of this chapter is the derivation of the other two price expectations hypotheses-adaptive and cash-futures price expectations.

## Adaptive Price Expectations of Soybean Market

We will use the formation as the following:

$$
P_{t}^{e}=a \sum_{i=0}^{\infty}(1-a)^{i} P_{t-i-1} ; 0<a<1
$$

Applying the concept to (4.47) through (4.50), equation (4.47) can be written as:

$$
\begin{aligned}
A_{t+4}= & A_{t}-\frac{1}{d 1} \frac{\alpha 1}{1-b^{4}}+\frac{1}{d 1} b^{2} y \beta \sum_{k=0}^{\infty}(b)^{4 k}\left(P S_{t+6+4 k}^{e}\right) \\
& +\frac{1}{d 1} b^{2} y \sum_{k=0}^{\infty}(b)^{4 k}\left(s_{t+6+4 k}^{e}\right)
\end{aligned}
$$

Let assume that $P S_{t+6+4 k}^{e}=g \sum_{j=1}^{\infty}(1-g)^{j} P S_{t+5+4 k-j}$; and

$$
\begin{gathered}
s_{t+6+4 k}^{e}=h \sum_{j=1}^{\infty}(1-h)^{j} s_{t+5+4 k-j} ;(4 k-j) \leq 0 \\
\text { and } t=0,4,8, \ldots
\end{gathered}
$$

We can write $A_{t+4}$ as:

$$
\begin{aligned}
A_{t+4}= & A_{t}-\frac{1}{d 1} \frac{\alpha 1}{1-b^{4}}+\frac{1}{d 1} b^{2} y B \sum_{k=0}^{\infty}(b)^{4 k} g \sum^{\infty} \quad(1-g)^{j} P_{t+5+4 k-j} \\
& +\frac{1}{d 1} b^{2} y \sum_{k=0}^{\infty}(b)^{4 k} \sum_{j=0}^{\infty}(1-h)^{j} s_{t+5+4 k-j}
\end{aligned}
$$

$$
\begin{align*}
& A_{t+4}= A_{t}-\frac{1}{d 1\left(1-b^{4}\right)}+\frac{1}{d 1} b^{2} y g \beta \sum_{j=1}^{\infty} \sum_{k=0}^{\infty} b^{4 k}(1-g)^{j} P_{t+5+4 k-j} \\
&+\frac{1}{d 1} b^{2} y h \sum_{j=1}^{\infty} \sum_{k=0}^{\infty} b^{4 k}(1-h)^{j} s_{t+5+4 k-j}  \tag{4.64}\\
& \text { for } t=0,4,8, \ldots
\end{align*}
$$

Thus, the soybean acreage planted next season depends upon this season acreage planted, current and lagged values of the prices $\mathrm{PS}_{t}$ and $s_{t}$.

Soybean inventory on-farm can be written as:

$$
\begin{align*}
\text { SBHF }_{t+2}= & \rho 1 \operatorname{SBHF}_{t+1}+\frac{\rho 1}{d 4} \sum_{i=0}^{\infty}(\rho 1 b)^{i}\left(b\left(\beta P S_{t+3+1}^{e}+s_{t+3+i}^{e}\right)\right. \\
& \left.-\left(\beta P S_{t+2+i}^{e}+s_{t+2+i}^{e}\right)-d 2\right) \tag{4.65}
\end{align*}
$$

Let assume that $\operatorname{PS}_{t+1+3}^{e}=g \sum_{j=0}^{\infty}(1-g){ }^{j}{ }_{P S}^{t+i+2-j} 1 ;(i-j) \leq 0$

$$
s_{t+i+3}^{e}=h \sum_{j=0}^{\infty}(1-h)^{j} s_{t+i+2-j} ;(i-j) \leq 0
$$

Thus, $\mathrm{SBHF}_{\mathrm{t}+2}$ can be written as:

$$
\begin{align*}
\text { SBHF }_{t+2}= & \rho 1 S B H F_{t+1}+\frac{(\rho i)}{d 4} \sum_{i=0}^{\infty}(\rho I b)^{i}\left(b g \sum_{j=0}^{\infty}(1-g)^{j_{P S}}{ }_{t+i+2-j}\right. \\
& \left.-g \sum_{j=0}^{\infty}(1-g)^{j_{P S}}{ }_{t+i+1-j}\right) \\
& +\frac{\rho 1}{d 4} \sum_{j=0}^{\infty}(\rho 1 b)^{i}\left(b h \sum_{j=0}^{\infty}(1-h)^{j} S_{t+i+2-j}\right. \\
& \left.-h \sum_{j=0}^{\infty}(1-h)^{j} s_{t+i+1-j}\right)-\frac{\rho 1 d 2}{d 4(1-\rho l b)} ; \quad(i-j) \leq 0 \quad(4 \tag{4.66}
\end{align*}
$$

${ }^{7}$ Since $\sum_{k=0}^{\infty} b^{4 k}, \sum_{j=0}^{\infty}(1-g)^{j}$ and $\sum_{j=0}^{\infty}(1-h)^{j}$ are absolute summable, we can interchange the summations.

Equation (4.66) says that soybean inventory on-farm for the end of the harvesting period depends upon the inventory in $t+1$ period and the sum of the weighted average of the changes in the soybean prices and the soybean margins. The quantity of soybean crushed can be written as:

$$
\begin{align*}
\mathrm{SC}_{\mathrm{t}+2}= & \lambda 1 \mathrm{SC}_{\mathrm{t}+1}+\frac{\lambda 1}{\mathrm{gl}} \sum_{i=0}^{\infty}(\lambda 1 \delta)^{1}\left(\mathrm{PM}_{\mathrm{t}+\mathrm{i}+2} * \mathrm{SOMSC}+\mathrm{PO}_{\mathrm{t}+1+2}\right. \\
& \left.* \operatorname{SOOSC}-\mathrm{PS}_{\mathrm{t}+\mathrm{i}+2}\right) ; \text { or } \\
\mathrm{SC}_{\mathrm{t}+2}= & \lambda 1 \mathrm{SC}_{\mathrm{t}+1}+\frac{\lambda 1}{\mathrm{gl}} \operatorname{SOMSC} \sum_{i=0}^{\infty}(\lambda 1 \delta)^{i} \mathrm{PM}_{\mathrm{t}+\mathrm{i}+2}^{\mathrm{e}} \\
& +\frac{\lambda 1}{\mathrm{gI}} \operatorname{sOOSC} \sum_{i=0}^{\infty}(\lambda 1 \delta)^{i} \mathrm{PO}_{\mathrm{t}+2+1}^{\mathrm{e}} \\
& -\frac{\lambda 1}{\mathrm{gI}} \sum_{i=0}^{\infty}(\lambda 1 \delta)^{1} \mathrm{PS}_{\mathrm{t}+1+2}^{\mathrm{e}} \tag{4.67}
\end{align*}
$$

Let assume that $P M^{e} t+2+i=\eta \sum_{j=0}^{\infty}(1-\eta)^{j} P M_{t+i+1-j} ;(1-j) \leq 0$

$$
P O_{t+i+2}^{e}=\phi \sum_{j=0}^{\infty}(1-\phi)^{j} P 0_{t+i+1-j ;} ;(i-j) \leq 0
$$

$S C_{t+2}$, then is:

$$
\begin{aligned}
& S C_{t+2}=\lambda 1 S C_{t+1}+\frac{\lambda 1}{g 1} \operatorname{SOMSC} \sum_{i=0}^{\infty}(\lambda 1 \delta)^{i} \sum_{j=0}^{\infty}(1-\eta)^{j} P M_{t+1-j+1} \\
& +\frac{\lambda 1}{g 1} \operatorname{soosc} \sum_{i=0}^{\infty}(\lambda 1 \delta)^{i_{\phi}} \sum_{j=0}^{\infty}(1-\phi)^{j} P 0_{t+i+1-j} \\
& -\frac{\lambda 1}{g 1} \sum_{i=0}^{\infty} \cdot(\lambda 1 \delta)^{i} g \sum_{j=0}^{\infty}(1-g)^{j} P S_{t+i+1-j} \\
& \text { where }(i-j) \leq 0
\end{aligned}
$$

Equation (4.68) says that the quantity of soybeans crushed in the harvesting time depends upon the level of soybeans crushed and the past history of PM, PO, and PS. Without going through the same process we can also solve for the soybean inventory off-farm as:

$$
\begin{align*}
S B C_{t+2} & =\theta 1 S B C_{t+1}+\frac{\theta 1}{g 3} \sum_{i=0}^{\infty}(\theta 1 \delta)^{i}\left(\delta S_{t+i+3}-P_{t+i+2}\right) \\
= & \theta 1 S_{B C} C_{t+1}+\frac{\theta 1}{g 3} \sum_{i=0}^{\infty}(\theta 1 \delta)^{i}\left(\delta g \sum_{j=0}^{\infty}(1-g)^{j_{P S}}{ }_{t+i+2-j}\right. \\
& -g \sum_{j=0}^{\infty}(1-g)^{j} \dot{P S}_{t+i+1-j} ;(i-j) \leq 0 \tag{4.69}
\end{align*}
$$

It is worth noting that the set of equations under adaptive price expectations is very similar to the set of equations under the rational expectations hypothesis. The conventional practice of estimating the adaptive version is to regress these equations without the crossequation restrictions as in the rational expectations version. Using the Koyck transformation we can get rid of the infinite summation.

## Cash-Futures Price Expectations of Soybean Market

Under this hypothesis we will follow the Samuelson hypothesis on anticipated speculative prices (1965, 1971, and 1976). If $P_{t}$ is the sequence of spot prices of a commodity and the sequence of prices of futures contracts are ( $\left.\ldots, y_{\theta, t}, y_{\theta, t+1}, \ldots y_{\theta, \theta}=P\right)$ when $y_{\theta, t}$ is the price quoted at $t$ for a futures contracts delivery at $\theta$ (which is $\theta-\mathrm{t}$ periods ahead).

Using the Samuelson (1965) hypothesis we get:

$$
\begin{equation*}
y_{\theta, t}=E_{t}\left(P_{\theta} \mid P_{t}, P_{t-1}, \ldots\right) \tag{4.69}
\end{equation*}
$$

In words, the futures price quote at $t$ for delivery time at $\theta$ is the expected cash price to prevail at time $\theta$, given past values of the spot prices and current spot price. Variable $y$ is in fact a martingale series. Let $P^{\operatorname{PMF}} \mathrm{t}+i+1, \mathrm{t}$, $\mathrm{POF}_{\mathrm{t}+i+1}$, be futures prices of soybean meal and oil quote at $t$ for $t+i+1$ contracts.

$$
\begin{aligned}
\operatorname{PSF}_{t+i+1, t} & =a \text { futures price quoted at } t \text { for } t+i+1 \text { contract. } \\
\text { SBCMF }_{t+i+1, t} & =\operatorname{PMF}_{t+i+1, t} * \operatorname{SOMSC}+\text { POF }_{t+i+1, t} * \operatorname{SOOSC}-\operatorname{PSF}_{t+i+1, t}
\end{aligned}
$$

Futures prices of soybeans, soybean meal and soybean oil are used as proxies of their anticipated cash prices. Substituting PSF, PMF and POF into (4.47) through (4.50), a set of model equations under the cash-futures price expectations is obtained as follows:

$$
\begin{align*}
& A_{t+4}=A_{t}-\left(\frac{1}{d_{1}}\right)\left(\frac{\alpha_{1}}{1-b^{4}}\right)+\left(\frac{\beta}{d_{1}} * b^{2} y\right)\left[\sum_{k=0}^{\infty}(b)^{4 k_{P S F}}{ }_{t+6+4 k, t}\right] \\
& -\left(\frac{1}{d_{1}}\right) b^{2} y\left[\sum_{k=0}^{\infty}(b)^{k} s_{t+6+4 k}\right]  \tag{4.70}\\
& \text { SBHF }_{t+2}=\rho_{1} \text { SBHF }_{t+1}+\left(\frac{\rho_{1}{ }^{*}}{d_{4}}\right)(b-L) \sum_{i=0}^{\infty}\left(\rho_{1} b\right)^{i} \operatorname{PSF}_{t+i+3, t} \\
& +\left(\frac{\rho_{1}}{d_{4}}\right)(b-L) \sum_{i=0}^{\infty}\left(\rho_{1} b\right)^{i} s_{t+6+4 k}-\frac{\rho_{1} d_{2}}{d_{4}\left(1-\rho_{1} b\right)}  \tag{4.71}\\
& S C_{t+2}=\lambda_{1} S C_{t+1}+\frac{\text { somsc }}{20}\left(\frac{\lambda_{1}}{g_{1}}\right) \sum_{i=0}^{\infty}\left(\lambda_{1} \delta\right)^{i} P M F_{t+i+2, t} \\
& +\operatorname{soosc}\left(\frac{\lambda_{1}}{g_{1}}\right) \sum_{i=0}^{\infty}\left(\lambda_{1} \delta\right)^{i} \operatorname{POF}_{t+i+2, t} \\
& -\left(\frac{\lambda_{1}}{g_{1}}\right) \sum_{i=0}^{\infty}\left(\lambda_{1} \delta\right)^{i} \text { PSF }_{t+i+2, t}  \tag{4.72}\\
& S B C_{t+2}=\theta_{1} S B C_{t+1}+\left(\frac{\theta_{1}}{g_{3}}\right)(\delta-L) \sum_{i=0}^{\infty}\left(\theta_{1} \delta\right)^{1} P^{P S F}{ }_{t+i+3, t} \tag{4.73}
\end{align*}
$$

It is worth mentioning that further research is needed to improve this last hypothesis. With certain assumptions concerning the futures prices, we can
prove that this last hypothesis is, in fact, rational expectations. The second notice is that we can solve for the futures prices which clear the market. Cash prices can also be solved by applying the theory of storage cost. However, this will not be done in this research.

Notice that the appearances of the three sets of models are the same. However, the interpretations are quite different. The market price ( $\mathrm{PS}_{\mathrm{t}}$ ) is derived from the first-order conditions under the rational expectations, and all the decision rules subject to cross-equation restrictions, or all coefficients in the model are a nonlinear function of the structural parameters. In order to get asymptotically consistent estimates, we have to estimate all coefficients and free parameters together. This imposes a very complicated problem in estimation procedures.

Under the other two hypotheses, adaptive and cash-futures price expectations, econometricians do not have to impose the restrictions in the estimation, even though it is quite clear that all the coefficients under the other two alternatives also depend upon the structural parameters.

If we know the process of futures prices of soybeans, soybean meal and soybean oil, we can discover the underlying parameters $-d_{1}, d_{2}, d_{3}, d_{4}$, $g_{0}, g_{1}, g_{2}, g_{3}$ as in rational expectations. However, this cannot be done under adaptive expectations, because we cannot discover the "coefficient of expectations" in this version. Estimation of the model under rational expectations is presented in the next chapter.

CHAPTER 5. ESTIMATION OF THE SÓYBEAN MARKET

In Chapter four, a theoretical derivation of the farmer's and processor's decision rules was presented. Under the rational-expectations hypothesis, the set of derived decision rules are the optimal time paths for soybean acreage planted, inventory on-farm, soybeans crushed, and inventory off-farm. These decision rules reflect current and expected future profit opportunities for the agents. The decision-rule coefficients are highly nonlinear functions of the underlying parameters of the structural model. The underlying parameters which appear in the agent's objective functions $\left(\beta, d_{1}, d_{2}, d_{3}, d_{4}, g_{0}, g_{1}, g_{2}, g_{3}, b\right.$ and $\delta$ ) and parameters characterizing the processes of exogenous stochastic variables are to be estimated. The following exogenous variables are included:
i) exogenois variables which appear in objective functions, and
ii) variable(s) which help(s) to predict the variables in (i). Before discussing the estimation procedure, it is important to state the assumptions concerning the exogenous variables in (i) and (ii). Let $W(t)$ be a "causally prior" stochastic vector. The vector $W(t)$ is an observable vector of variables which is a subset of agents' information set, or,

$$
W(t) \subset \Omega(t) \text { and }
$$

$W(t) \cap \varepsilon(t)=0$, where $\varepsilon(t)$ is an unobservable variable.
If it is assumed that $Y(t)$ is a vector including choice variables -- $A_{t}$, $S B H F_{t}, S C_{t}$, and $S B C_{t}$ and endogenous price variables $\mathrm{PS}_{t}, \mathrm{PM}_{t}$ and $\mathrm{PO}_{t}$-- we require

$$
W(t) \cap Y(t)=0,
$$

or it implies that there are no choice variables or endogenous variables in vector $W(t)$. The last requirement is $Y(t)$ must fail to Granger-cause $W(t)$. If the Granger test is accepted, a finite one-sided distributed lag of $W(t)$ can be written on the right-hand side of $Y(t)$. Thus, Granger-causality tests must be performed for all variables before estimating the model.
U.S. quarterly data from 1962 through 1977 are used (although data from 1960 through 1961 are used as initial values). Most data are obtained from various statistical reports and publications of the United States Department of Agriculture, unless otherwise stated. Other data were obtained by personal contact with USDA officials and Iowa State University Collegues. ${ }^{1}$

## Estimation Procedures of the Soybean Market Under the Rational Expectations

The objective of this sector is to estimate the model's parameters using all available information and cross-equation restrictions in the model derived in Chapter four. There are three steps needed for this estimation. The first step is to adjust all data by their means and time trend. Then, this adjusted data is used to perform stationary tests using Fuller's method (Fuller, 1976). Some literature suggests taking. seasonality as well as trend out of the series. This is debatable.

[^2]Whether or not to remove seasonality depends upon the purpose of the research. It is believed that seasonality cannot be removed independently from the several series being dealt with in this research because of the seasonal interrelationships which exist among various series in this model (Kallek, 1978). Granger (1978) also called attention to the causes of seasonality, Among other things, weather and expectations are at least two factors which cause seasonality in agricultural commodities such as soybeans. Though modelling seasonality behavior is a very interesting area, it is a separate subject from this research.

The results from Fuller's test can be used to determine the lag length of the realization of all time series. The second step is to perform the Granger-causality test on the following pair of vectors:

$$
\begin{aligned}
& Y(t)+W(t) \\
& W(t)+Y(t)
\end{aligned}
$$

The method used for causality test is Sargent (1979), which uses the Fstatistic.

The third step is to estimate equations (4.60), (4.61), (4.62) and (4.63) both with and without restrictions on the parameters. The details of this step are presented later.

## Time series analysis

The identification stage of the Box-Jenkins procedure has been performed for all raw data using the TSERIES package (Meeker, 1977). Autocorrelation functions of all variables indicate nonstationary time series and some series such as soybean stock and soybean acreage planted
show strong seasonality patterns. Therefore, Fuller's procedure is used to search for the "unit root" in, the time series.

Let $Z(t)$ be a $p^{\text {th }}$ order autoregressive time series such as:

$$
\begin{equation*}
Z(t)+\sum_{j=1}^{p} \alpha_{j} Z(t-j)=e(t) \tag{5.1}
\end{equation*}
$$

If a unit root is suspected, the unit root should be isolated as a coefficient and a test performed. ${ }^{2}$ Equation (5.1) can be written as:

$$
\begin{gather*}
Y_{t}=\theta_{1} Y_{t-1}+\sum_{j=2}^{p} \theta_{j}\left(Y_{t-j+1}-Y_{t-j}\right)+e_{t} \\
\text { for } t=p+1, p+2, \cdots \tag{5.2}
\end{gather*}
$$

where $p \geq 2, \theta_{i}=\sum_{j=i}^{p} \alpha_{j}, i=2,3, \ldots p$, and $\theta_{1}=\sum_{j=1}^{p} \alpha_{j}$. If there is a unit root, $\theta_{1}$ is equal to one. Thus, this test is applied to all adjusted data with various orders of $p$. . By using the cumulative distribution in Fuller (1976, p. 373), the hypothesis of $\theta_{1}$ being one can be tested. Table B. 1 and B. 2 in Appendix B give results of the test for adjusted endogenous and exogenous variables. Various orders of autoregressive models are tested; however, only those which are significant or those which give small mean square errors are reported. The significance of the fourth and fifth order autoregression terms indicate the potential of seasonality in the series. Examples of such series are soybean stock, soybean acreage planted, soybean oil stock, soybean meal stock, fish meal price, and high
${ }^{2}$ The proof of this test is in Fuller (1976, pp. 366-382). A test of a unit root for moving average models can also be found in Fuller.
protein animal units. The rejection of null hypothesis of $\theta_{1}$ equals one in the second-order test for, say, $\mathrm{PS}_{t}$, implies that the second-order autoregressive model is a good candidate to be used for the realization of $\mathrm{PS}_{t}$. When more than one realization is accepted, further tests and adjustments are needed to conclude the test. Table B3 is the "t-like" distribution for the test. The critical statistic value for infinite degrees of freedom at $\alpha=0.05$ is -3.41 . We reject the null hypothesis of $\theta_{1}$ being one if the computed "t-like" statistic is greater than the critical value in absolute value or $\hat{\tau}=\left|\frac{\hat{\theta}_{1}-1}{s \hat{\theta}_{1}}\right|>t_{0.05}$. The lagged length from this test is used in estimating the autoregressive model.

The results from Table B. 1 and B. 2 confirm that some data have strong seasonality. Therefore, autoregressive models of order two to six have been estimated as the following:

$$
\begin{align*}
Y(t)= & C+\sum_{s=1}^{6} a(s) Y(t-s)+\gamma_{1} \text { Fall } \\
& +\gamma_{2} \text { Wint }+\gamma_{3} \text { Spri }+e_{t}  \tag{5.3}\\
\text { and, } X(t)= & C+\sum_{s=1}^{6} \beta(s) X(t-s)+\gamma_{1} \text { Fall } \\
& +\gamma_{2} \text { Wint }+\gamma_{3} \operatorname{Spri}+e_{t} \tag{5.4}
\end{align*}
$$

where $Y(t)$ and $X(t)$ are all endogenous and exogenous variables, and Fall, Wint and Spri are seasonal dummies for the fall, winter and spring quarters. The results of the autoregressive models (AR) are reported in Tables 5.1.1 to 5.1.2. The models which represent the realizations of all variables are summarized in Table 5.1.3.

Table 5.1.1. Autoregressive model of detrend endogenous variables with seasonal dummy variables.

| Mod | $Y(t)=C+$ | $\sum_{s=1}^{8} \alpha(s) Y(t-s)+\gamma_{1} \text { Fall }+\gamma_{2} \text { Wint }+\gamma_{3} \text { Spri }+e_{t}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\alpha$ (2) | $\alpha$ (3) | $\alpha$ (4) | $\alpha$ (5) | $\alpha$ (8) |
| $A_{t}^{a}$ |  |  |  | $\begin{gathered} \mathrm{t} .05 \\ (0.01)^{\mathrm{b}} \end{gathered}$ |  |  |
| $A_{t}$ |  |  |  | $\begin{gathered} 0.85 \\ (0.13) \end{gathered}$ |  | $\begin{gathered} 0.22 \\ (0.13) \end{gathered}$ |
| PS ${ }_{t}$ | $\begin{gathered} 0.84 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.13 \\ & (0.13) \end{aligned}$ |  |  |  |  |
| $\mathrm{PS}_{t}$ | $\begin{gathered} 0.85 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.21 \\ & (0.17) \end{aligned}$ | $\begin{gathered} 0.09 \\ (0.13) \end{gathered}$ |  |  |  |
| PS ${ }_{t}$ | $\begin{gathered} 0.72 \\ (0.09) \end{gathered}$ |  |  | $\begin{aligned} & -0.04 \\ & (0.14) \end{aligned}$ | $\begin{gathered} 0.12 \\ (0.13) \end{gathered}$ |  |
| $\mathrm{SC}_{\mathrm{t}}$ | $\begin{gathered} 0.79 \\ (0.12) \end{gathered}$ | $\begin{aligned} & -0.36 \\ & (0.13) \end{aligned}$ |  |  |  |  |
| $\mathrm{SC}_{\mathrm{t}}$ | $\begin{gathered} 0.75 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.28 \\ & (0.17) \end{aligned}$ | $\begin{aligned} & -0.11 \\ & (0.13) \end{aligned}$ |  |  |  |
| SC ${ }_{\text {t }}$ | $\begin{gathered} 0.57 \\ (0.11) \end{gathered}$ |  |  | $\begin{aligned} & -0.14 \\ & (0.14) \end{aligned}$ | $\begin{gathered} 0.15 \\ (0.16) \end{gathered}$ |  |
| $S B C C_{t}$ | $\begin{gathered} 0.21 \\ (0.11) \end{gathered}$ |  |  |  |  |  |
| $\mathrm{SBC}_{\mathrm{t}}$ | $\begin{gathered} 0.20 \\ (0.12) \end{gathered}$ | $\begin{gathered} 0.008 \\ (0.12) \end{gathered}$ |  |  |  |  |
| $S B C C_{t}$ | $\begin{gathered} 0.37 \\ (0.11) \end{gathered}$ |  |  | $\begin{gathered} 0.56 \\ (0.09) \end{gathered}$ | $\begin{aligned} & -0.37 \\ & (0.11) \end{aligned}$ |  |

[^3]| Fall 1 | Wint | Spri | MSE | $\mathrm{R}^{2}$ | $\mathrm{F}_{5,74}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 3.44 | 0.99 | 6254* |
|  |  |  | 0.36 | 0.99 | 3208* |
| $\begin{aligned} & -0.39 \\ & (0.18) \end{aligned}$ | $\begin{gathered} 0.05 \\ (0.19) \end{gathered}$ | $\begin{gathered} 0.20 \\ (0.18) \end{gathered}$ | 0.68 | 0.60 | 14.83* |
| $\begin{aligned} & -0.38 \\ & (0.19) \end{aligned}$ | $\begin{gathered} 0.07 \\ (0.19) \end{gathered}$ | $\begin{gathered} 0.14 \\ (0.19) \end{gathered}$ | 0.68 | 0.57 | 12.33* |
| $\begin{aligned} & -0.41 \\ & (0.19) \end{aligned}$ | $\begin{gathered} 0.01 \\ (0.19) \end{gathered}$ | $\begin{gathered} 0.23 \\ (0.18) \end{gathered}$ | 0.69 | 0.60 | 12.14* |
| $\begin{gathered} 6.68 \\ (3.23) \end{gathered}$ | $\begin{gathered} 6.74 \\ (3.03) \end{gathered}$ | $\begin{aligned} & -3.02 \\ & (3.03) \end{aligned}$ | 161. | 0.52 | 11.96* |
| $\begin{array}{r} 6.87 \\ (3.2) \end{array}$ | $\begin{gathered} 7.89 \\ (3.35) \end{gathered}$ | $\begin{aligned} & -3.54 \\ & (3.11) \end{aligned}$ | 162.6 | 0.52 | 10.01* |
| $\begin{gathered} 4.13 \\ (3.4) \end{gathered}$ | $\begin{aligned} & 11.34 \\ & (3.3) \end{aligned}$ | $\begin{aligned} & -1.03 \\ & (3.08) \end{aligned}$ | 185.2 | 0.45 | 7.67* |
| $\begin{aligned} & 288.14 \\ & (30.3) \end{aligned}$ | $\begin{array}{r} 26.19 \\ (.31 .3) \end{array}$ | $\begin{aligned} & -99.24 \\ & (16.9) \end{aligned}$ | 5387.0 | 0.86 | 108.2* |
| $\begin{aligned} & 288.4 \\ & (30.7) \end{aligned}$ | $\begin{gathered} 28.5 \\ (46.0) \end{gathered}$ | $\begin{aligned} & -101.0 \\ & (31.5) \end{aligned}$ | 5465.0 | 0.86 | $85.4 *$ |
| $\begin{aligned} & 115.18 \\ & (36.5) \end{aligned}$ | $\begin{array}{r} 29.57 \\ (27.7) \end{array}$ | $\begin{gathered} -39.8 \\ (16.5) \end{gathered}$ | 3476.0 | 0.91 | 118.61* |

Table 5.1.1 - continued

| Model: | $Y(t)=C+\underset{s}{6}$ |  |  | $\gamma_{1}$ Fall $+\gamma_{2}$ Wint $+\gamma_{3}$ Spri $+e_{t}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | C | (1) | (2) | (3) | (4) | (5) | (6) |
| $S B C_{t}$ | $\begin{aligned} & -2.54 \\ & (7.0) \end{aligned}$ | $\begin{gathered} 0.37 \\ (0.12) \end{gathered}$ | $\begin{aligned} & -0.007 \\ & (0.12) \end{aligned}$ |  | $\begin{gathered} 0.55 \\ (0.09) \end{gathered}$ | $\begin{aligned} & -0.36 \\ & (0.11) \end{aligned}$ | $\begin{aligned} & -0.03 \\ & (0.12) \end{aligned}$ |
| $\mathrm{SBHF}_{\mathrm{t}}$ | $\begin{aligned} & -5.93 \\ & (9.4) \end{aligned}$ | $\begin{gathered} 0.27 \\ (0.11) \end{gathered}$ |  |  |  |  |  |
| $\mathrm{SBHF}_{\mathrm{t}}$ | $\begin{gathered} -1.28 \\ (7.2) \end{gathered}$ | $\begin{gathered} 0.55 \\ (0.11) \end{gathered}$ |  |  | $\begin{gathered} 0.69 \\ (0.09) \end{gathered}$ | $\begin{aligned} & -0.43 \\ & (0.11) \end{aligned}$ |  |
| $\mathrm{SBHF}_{\mathrm{t}}$ | $\begin{aligned} & -1.1 \\ & (6.9) \end{aligned}$ | $\begin{gathered} 0.79 \\ (0.12) \end{gathered}$ | $\begin{aligned} & -0.40 \\ & (0.12) \end{aligned}$ |  | $\begin{gathered} 0.79 \\ (0.09) \end{gathered}$ | $\begin{aligned} & -0.69 \\ & (0.14) \end{aligned}$ | $\begin{gathered} 0.28 \\ (0.12) \end{gathered}$ |
| $\mathrm{PM}_{t}$ | $\begin{aligned} & -2.16 \\ & (3.1) \end{aligned}$ | $\begin{gathered} 0.97 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.29 \\ & (0.13) \end{aligned}$ |  |  |  |  |
| $\mathrm{PM}_{t}$ | $\begin{aligned} & -2.27 \\ & (3.12) \end{aligned}$ | $\begin{gathered} 0.95 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.22 \\ & (0.18) \end{aligned}$ | $\begin{aligned} & -0.07 \\ & (0.13) \end{aligned}$ |  |  |  |
| $\mathrm{PO}_{t}$ | $\begin{aligned} & -0.1 \\ & (0.44) \end{aligned}$ | $\begin{gathered} 0.88 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.05 \\ & (0.13) \end{aligned}$ |  |  |  |  |
| $\mathrm{PO}_{t}$ | $\begin{gathered} -0.16 \\ (0.43) \end{gathered}$ | $\begin{gathered} 0.87 \\ (0.08) \end{gathered}$ |  |  | $\begin{aligned} & .0 .12 \\ & (0.14) \end{aligned}$ | $\begin{aligned} & -0.24 \\ & (0.13) \end{aligned}$ |  |


| Fa11 | Wint | Spri | MSE | $\mathrm{R}^{2}$ | $\mathrm{~F}_{5.74}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 113.1 | 20.7 | -31.6 | 3574.0 | 0.91 | $86.52^{*}$ |
| $(37.5)$ | $(39.5)$ | $(29.6)$ |  |  |  |
| 286.5 | -14.3 | -111.3 | 6561.0 | 0.81 | $74.9^{*}$ |
| $(26.5)$ | $(31.5)$ | $(17.1)$ |  |  |  |
| 103.6 | -13.4 | -36.51 | 3812.0 | 0.89 | $94.7^{*}$ |
| $(32.7)$ | $(26.8)$ | $(16.68)$ |  |  |  |
| 67.9 | -39.7 | 3.77 | 3380.0 | 0.91 | $81.47^{*}$ |
| $(33)$. | $(32.9)$ | $(28.5)$ |  |  |  |
| -6.57 | 4.89 | 3.13 | 573.6 | 0.61 | $17.61^{*}$ |
| $(5.3)$ | $(5.4)$ | $(5.24)$ |  |  |  |
| -6.84 | 4.86 | 3.64 | 580.9 | 0.61 | $14.53^{*}$ |
| $(5.39)$ | $(5.44)$ | $(5.36)$ |  |  |  |
| -0.93 | -0.09 | 0.05 | 11.78 | 0.71 | $26.79^{*}$ |
| $(0.77)$ | $(0.77)$ | $(0.75)$ |  |  |  |
| -0.72 | -0.15 | 0.05 | 10.98 | 0.73 | $24.81^{*}$ |
| $(0.75)$ | $(0.74)$ | $(0.72)$ |  |  |  |

Table 5.1.2. Autoregressive model of detrend exogenous variables with seasonal dummies

| Model: | $=X_{0}+\sum_{s=1}^{6} \beta(s) X(t-s)+\gamma_{1} \text { Fall }+\gamma_{2} \text { Wint }+\gamma_{3} \text { Spri }+e_{t}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | C | $\beta$ (1) | B(2) | $\beta$ (3) | B(4) | $\beta$ (5) | $\beta(6)$ |
| $\mathrm{SBX}_{\mathrm{t}}$ | $\begin{aligned} & -6.8 \\ & (2.9) \end{aligned}$ | $\begin{gathered} 0.06 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.05 \\ & (0.14) \end{aligned}$ |  |  |  |  |
| SBX ${ }_{t}$ | $\begin{aligned} & -3.7 \\ & (2.9) \end{aligned}$ | $\begin{gathered} 0.11 \\ (0.14) \end{gathered}$ |  |  | $\begin{gathered} 0.30 \\ (0.14) \end{gathered}$ | $\begin{gathered} 0.01 \\ (0.15) \end{gathered}$ |  |
| SBX ${ }_{t}$ | $\begin{aligned} & -8.09 \\ & (3.03) \end{aligned}$ | $\begin{gathered} 0.05 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.04 \\ & (0.14) \end{aligned}$ | $\begin{aligned} & -0.18 \\ & (0.14) \end{aligned}$ |  |  |  |
| $\mathrm{SBX}_{\mathrm{t}}$ | $\begin{aligned} & -6.1 \\ & (2.7) \end{aligned}$ |  |  | $\begin{aligned} & -0.21 \\ & (0.13) \end{aligned}$ | $\begin{gathered} 0.30 \\ (0.13) \end{gathered}$ |  |  |
| sox | $\begin{aligned} & -6.4 \\ & (9.5) \end{aligned}$ | $\begin{gathered} 0.85 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.17 \\ & (0.13) \end{aligned}$ |  |  |  |  |
| sox | $\begin{aligned} & -6.02 \\ & (9.6) \end{aligned}$ | $\begin{gathered} 0.86 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.22 \\ & (0.17) \end{aligned}$ | $\begin{gathered} 0.06 \\ (0.13) \end{gathered}$ |  |  |  |
| sox | $\begin{aligned} & -7.6 \\ & (9.5) \end{aligned}$ | $\begin{gathered} 0.73 \\ (0.09) \end{gathered}$ |  |  | $\begin{gathered} 0.14 \\ (0.12) \end{gathered}$ | $\begin{aligned} & -0.24 \\ & (0.12) \end{aligned}$ |  |
| SOH | $\begin{aligned} & -17.3 \\ & (13.7) \end{aligned}$ | $\begin{gathered} 1.18 \\ (0.12) \end{gathered}$ | $\begin{aligned} & -0.39 \\ & (0.12) \end{aligned}$ |  |  |  |  |
| SOH | $\begin{aligned} & -18.8 \\ & (13.4) \end{aligned}$ | $\begin{gathered} 1.09 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.09 \\ & (0.19) \end{aligned}$ | $\begin{aligned} & -0.24 \\ & (0.13) \end{aligned}$ |  |  |  |
| SOH | $\begin{aligned} & -19.12 \\ & (13.4) \end{aligned}$ | $\begin{gathered} 0.98 \\ (0.07) \end{gathered}$ |  |  | $\begin{aligned} & -0.44 \\ & (0.13) \end{aligned}$ | $\begin{gathered} 0.23 \\ (0.12) \end{gathered}$ |  |
| SOH | $\begin{aligned} & -19.3 \\ & (13.4) \end{aligned}$ | $\begin{gathered} 1.13 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.22 \\ & (0.17) \end{aligned}$ |  | $\begin{aligned} & -0.39 \\ & (0.17) \end{aligned}$ | $\begin{array}{r} 0.39 \\ (0.2) \end{array}$ | $\begin{aligned} & -0.16 \\ & (0.14) \end{aligned}$ |
| SMH | $\begin{aligned} & 1.87 \\ & (5.7) \end{aligned}$ | $\begin{gathered} 1.1 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.27 \\ & (0.13) \end{aligned}$ |  |  |  |  |


| Fall | Wint | Spri | MSE | $\mathrm{R}^{2}$ | F |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & 14.31 \\ & (6.0) \end{aligned}$ | $\begin{array}{r} 6.09 \\ (6.3) \end{array}$ | $\begin{gathered} 8.73 \\ (4.87) \end{gathered}$ | 397.7 | 0.45 | 9.13* |
| $\begin{aligned} & 11.8 \\ & (6.5) \end{aligned}$ | $\begin{gathered} 4.2 \\ (4.9) \end{gathered}$ | $\begin{gathered} 5.7 \\ (4.5) \end{gathered}$ | 372.8 | 0.49 | 8.91* |
| $\begin{aligned} & 15.57 \\ & (6.04) \end{aligned}$ | $\begin{aligned} & 7.56 \\ & (6.4) \end{aligned}$ | $\begin{gathered} 3.6 \\ (6.2) \end{gathered}$ | 392.6 | 0.46 | 8.0* |
| $\begin{aligned} & 10.24 \\ & (4.6) \end{aligned}$ | $\begin{array}{r} 6.91 \\ (4.4) \end{array}$ | $\begin{aligned} & 1.06 \\ & (5.6) \end{aligned}$ | 355.5 | 0.51 | 11.55* |
| $\begin{aligned} & -20.77 \\ & (17.12) \end{aligned}$ | $\begin{gathered} 13.34 \\ (16.6) \end{gathered}$ | $\begin{gathered} 48.09 \\ (16.05) \end{gathered}$ | 5,290.4 | 0.57 | 15.05* |
| $\begin{gathered} -17.52 \\ (18.5) \end{gathered}$ | $\begin{array}{r} 11.19 \\ (17.3) \end{array}$ | $\begin{array}{r} 46.26 \\ (16.6) \end{array}$ | 5,363.8 | 0.57 | 12.41* |
| $\begin{aligned} & -22.7 \\ & (16.6) \end{aligned}$ | $\begin{array}{r} 4.64 \\ (16.5) \end{array}$ | $\begin{gathered} 40.9 \\ (17.4) \end{gathered}$ | 5,170.7 | 0.60 | 13.22* |
| $\begin{aligned} & 118.4 \\ & (26.3) \end{aligned}$ | $\begin{gathered} -8.9 \\ (25.6) \end{gathered}$ | $\begin{aligned} & -14.9 \\ & (22.9) \end{aligned}$ | 10,782. | 0.79 | 41.69* |
| $\begin{aligned} & 113.74 \\ & (25.8) \end{aligned}$ | $\begin{array}{r} 21.43 \\ (29.8) \end{array}$ | $\begin{aligned} & -34.02 \\ & (24.6) \end{aligned}$ | 10,323. | 0.80 | 36.86* |
| $\begin{aligned} & 115.6 \\ & (26.9) \end{aligned}$ | $\begin{gathered} 41.9 \\ (23.07) \end{gathered}$ | $\begin{aligned} & -10.04 \\ & (22.6) \end{aligned}$ | 10,279. | 0.80 | 37.06* |
| $\begin{aligned} & 148.4 \\ & (34.4) \end{aligned}$ | $\begin{array}{r} 6.15 \\ (32.5) \end{array}$ | $\begin{aligned} & -20.7 \\ & (23.6) \end{aligned}$ | 10,191. | 0.81 | 28.35* |
| $\begin{array}{r} 24.32 \\ (11.3) \end{array}$ | $\begin{gathered} 10.2 \\ (10.2) \end{gathered}$ | $\begin{gathered} 1.74 \\ (9.9) \end{gathered}$ | 1,997.4 | 0.78 | 40.02* |

Table 5.1.2-continued

| Mode1: | $\sum_{s=1}^{6} \beta(s) X(t-s)+\gamma_{1}$ Fall $+\gamma_{2}$ Wint $+\gamma_{3}$ Spri $+e_{t}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | C | $\beta(1)$ | $\beta(2)$ | $B(3)$ | $\beta(4)$ | $\beta(5)$ | $B(6)$ |
| SMH | $\begin{array}{r} 1.88 \\ (5.8) \end{array}$ | $\begin{gathered} 1.1 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.26 \\ & (0.21) \end{aligned}$ | $\begin{aligned} & -0.005 \\ & (0.14) \end{aligned}$ |  |  |  |


| SMH | 1.6 | 0.98 | -0.37 | 0.24 |
| :---: | :---: | :---: | :---: | :---: |
|  | $(5.7)$ | $(0.07)$ | $(0.15)$ | $(0.14)$ |


| HPAV | -0.05 | 0.82 | -0.04 |
| :---: | :---: | :---: | :---: |
|  | $(0.07)$ | $(0.13)$ | $(0.13)$ |


| HPAV | -0.04 | 0.82 | -0.12 | 0.10 |
| :---: | :---: | :---: | :---: | :---: |
|  | $(0.08)$ | $(0.13)$ | $(0.16)$ | $(0.12)$ |


| HPAV | -0.05 | 0.79 |  | 0.156 | -0.21 |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $(0.07)$ | $(0.09)$ |  | $(0.13)$ | $(0.12)$ |  |
|  |  |  |  | 0.16 | -0.29 | 0.104 |
| HPAV | -0.05 | 0.83 | -0.04 | $(0.13)$ | $(0.16)$ | $(0.12)$ |


| C00P | 0.07 | 0.58 | 0.23 |
| :---: | :---: | :---: | :---: |
|  | $(0.45)$ | $(0.13)$ | $(0.13)$ |


| COOP | -0.07 | 0.58 | 0.23 | -0.01 |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $(0.45)$ | $(0.14)$ | $(0.15)$ | $(0.16)$ |  |  |
| COOP | -0.10 | 0.92 |  |  | -0.47 | 0.29 |
|  | $(0.43)$ | $(0.1)$ |  |  | $(0.15)$ | $(0.13)$ |

FIMP $_{\mathrm{t}}$|  | -1.64 | 1.11 | -0.34 |
| :---: | :---: | :---: | :---: |
|  | $(4.95)$ | $(0.13)$ | $(0.13)$ |

| FIMP $_{\mathrm{t}}$ | -1.7 | 1.05 | -0.13 | -0.18 |
| :--- | :--- | :---: | :---: | :---: |
|  | $(4.9)$ | $(0.13)$ | $(0.19)$ | $(0.13)$ |

FIMP $_{\mathrm{t}} \quad$| -2.07 | 0.89 |  |
| :---: | :---: | :---: |
|  | $(4.66)$ | $(0.08)$ |

| -0.2 | -0.08 |
| :--- | :--- |
| $(0.14)$ | $(0.13)$ |


| Fall | Wint | Spri | MSE | $\mathrm{R}^{2}$ | F |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 24.2 | 10.4 | 1.68 | 2,033.7 | 0.78 | 32.75* |
| (11.6) | (12.02) | (10.24) |  |  |  |
| 18.98 | 21.85 | 8.98 | 1,942 | 0.79 | 34.72* |
| (10.2) | (10.2) | (9.7) |  |  |  |
| 3.51 | -1.53 | -3.26 | 0.39 | 0.93 | 159.2* |
| (0.28) | (0.49) | (0.31) |  |  |  |
| 3.2 | -1.34 | -2.92 | 0.39 | 0.93 | 132.10* |
| (0.46) | (0.54) | (0.51) |  |  |  |
| 2.96 | -0.96 | -2.77 | 0.37 | 0.93 | 137.18* |
| (0.51) | (0.37) | (0.47) |  |  |  |
| 3.08 | -0.76 | -2.90 | 0.38 | 0.94 | 100.63* |
| (0.64) | (0.66) | (0.63) |  |  |  |
| -0.87 | 0.22 | 0.67 | 12.4 | 0.57 | 15.25* |
| (0.79) | (0.79) | (0.77) |  |  |  |
| -0.88 | 0.22 | 0.68 | 12.65 | 0.58 | 12.48* |
| (0.79) | (.80) | (0.78) |  |  |  |
| -1.07 | 0.6 | 0.79 | 11.32 | 0.62 | 15.02* |
| (0.75) | (0.75) | (0.74) |  |  |  |
| $\begin{array}{r} 6.31 \\ (8.7) \end{array}$ | 3.99 | -4.33 | 1,509.6 | 0.73 | 29.7* |
|  | (8.7) | (8.5) |  |  |  |
| $\begin{gathered} 6.9 \\ (8.7) \end{gathered}$ | 5.06 | -4.97 | 1,485.7 | 0.73 | 25.46* |
|  | (8.7) | (8.5) |  |  |  |
| 4.65 | 6.67 | -2.73 | 1,336. | 0.76 | 29.31* |
| (8.19) | (8.17) | (8.01) |  |  |  |

Table 5.1.2 - continued

| Model: | $\sum_{\mathrm{s}=1}^{6} \beta(\mathrm{~s}) \mathrm{X}(\mathrm{t}-\mathrm{s})+\gamma_{1} \text { Fall }+\gamma_{2} \text { Wint }+\gamma_{3} \text { Spri }+e_{t}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | C | $B(1)$ | B(2) | $B(3)$ | $\beta$ (4) | $\beta(5)$ | $B(6)$ |
| TB | $\begin{gathered} 0.62 \\ (0.26) \end{gathered}$ | $\begin{gathered} 1.25 \\ (0.12) \end{gathered}$ | $\begin{aligned} & -0.36 \\ & (0.12) \end{aligned}$ |  |  |  |  |
| TB | $\begin{gathered} 0.60 \\ (0.27) \end{gathered}$ | $\begin{gathered} 1.27 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.42 \\ & (0.21) \end{aligned}$ | $\begin{gathered} 0.05 \\ (0.13) \end{gathered}$ |  |  |  |
| CORPF | -0.008 | $\begin{gathered} 0.89 \\ (0.06) \end{gathered}$ |  |  |  |  |  |
| CORPF | -0.004 | $\begin{gathered} 0.97 \\ (0.07) \end{gathered}$ |  |  |  | $\begin{aligned} & -0.15 \\ & (0.07) \end{aligned}$ |  |
| CORPF | -0.003 | $\begin{aligned} & 0.87 \\ & (0.089) \end{aligned}$ |  | $\begin{gathered} 0.2 \\ (0.12) \end{gathered}$ |  | $\begin{aligned} & -0.25 \\ & (0.09) \end{aligned}$ |  |


| Fall | Wint | Spri | MSE | $\mathrm{R}^{2}$ | F |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & -0.08 \\ & (0.12) \end{aligned}$ | $\begin{aligned} & -0.21 \\ & (0.11) \end{aligned}$ | $\begin{aligned} & -0.05 \\ & (0.11) \end{aligned}$ | 0.28 | 0.86 | 72.19* |
| $\begin{aligned} & -0.09 \\ & (0.13) \end{aligned}$ | $\begin{aligned} & -0.19 \\ & (0.12) \end{aligned}$ | $\begin{aligned} & -0.05 \\ & (0.12) \end{aligned}$ | 0.29 | 0.87 | 59.25* |
|  |  |  | 0.04 | 0.79 |  |
|  |  |  | 0.04 | 0.80 |  |
|  |  |  | 0.03 | 0.81 |  |

Table 5.1.3. The selected autoregressive model

| Variables | Mode1 | Restriction |
| :---: | :---: | :---: |
| A | AR (8) | $\begin{aligned} \alpha(1) & =\alpha(2)=\alpha(3)=\alpha(5) \\ & =\alpha(6)=\alpha(7)=0 \end{aligned}$ |
| PS | AR(2) |  |
| SC | AR(2) |  |
| SBC | AR(5) | $\alpha(2)=\alpha(3)=0$ |
| PM | $\operatorname{AR}(2)$ |  |
| PO | AR(2) |  |
| SBHF | $\operatorname{AR}(6)$ | $\alpha(3)=0$ |
| SBX | AR(4) | $\beta(1)=\beta(2)=0$ |
| SOX | AR(2) |  |
| SOH | $\operatorname{AR}(5)$ | $\beta(2)=\beta(3)=0$ |
| SMH | AR (5) | $\beta(2)=\beta(3)=0$ |
| COOP | AR (2) |  |
| FIMP | AR(5) | $\beta(2)=\beta(3)=0$ |
| TB | AR(2) |  |
| hpad | AR(5) | $\alpha(2)=\alpha(3)=0$ |

Soybean stock on-farm (SBHF), soybean stock off-farm (SBC) and highprotein animal units are the most interesting series of all. For example, the SBC series has the following realization:

$$
S B C_{t}=0.37 \mathrm{SBC}_{t-1}+0.56 \mathrm{SBC}_{t-4}-0.37 \mathrm{SBC}_{t-5}+\mathrm{e}_{\mathrm{t}}^{3}
$$

Using Box-Jenkins notation, $S B C_{t}$ can be written as:

$$
\left(1-\theta_{1}\right)\left(1-\theta_{4}\right) S B C_{t}=e_{t},
$$

where $\theta_{1}$ is the coefficient for $S B C_{t-1}$, and $\theta_{4}$ is the coefficient for $S_{t-4}$, or, it can be written as:

$$
\begin{gathered}
\left(1-\theta_{1}-\theta_{4}+\theta_{1} \theta_{4}\right) S B C_{t}=e_{t} \text {, or } \\
S B C_{t}-\theta_{1} S B C_{t-1}-\theta_{4} S B C_{t-4}+\theta_{1} \theta_{4} S B C_{t-5}=e_{t}
\end{gathered}
$$

This is a first-order seasonal autoregressive process. The seasonality is presented in a multiplicative way. The coefficient " 0.37 " for $S_{t-5}$ is quite close to the multiplication of " 0.37 and 0.56 ". The series $S B H F_{t}$ has the second-order seasonal autoregressive process such as:

$$
\begin{gathered}
\left(1-\theta_{1}-\theta_{2}\right)\left(1-\theta_{4}\right) S B H F_{t}=\epsilon_{t} \text {, or } \\
\text { SBHF }_{t}-\theta_{1} S B H F_{t-1}-\theta_{2} S B H F_{t-2}-\theta_{4} S^{S B H F}{ }_{t-4}+\theta_{1} \theta_{4} S B H F_{t-5}+\theta_{2} \theta_{4} S B H F_{t-6}=e_{t}
\end{gathered}
$$ The interaction terms $\theta_{1} \theta_{4}$ and $\theta_{2} \theta_{4}$ make the analysis on the soybean stock very complicated. Explanation of seasonal time series is in Appendix $C$.

## Granger causality test

The variables used as candidates for the "causal" variables $Y$ are: corn price received by farmers
(CORPF)
$3^{3}$ The constant and seasonal terms are ignored for now.

| soybean export | (SBX) |
| :--- | ---: |
| soybean oil export | (SOX) |
| high-protein animal units | (HPAU) |
| cottonseed oil price received by farmers | (COOP) |
| soybean oil stock | (SOH) |
| soybean meal stock | (SMH) |
| T-bill interest rates | (TB) |
| fish meal price | (FIMP) | These variables are contained in vector $W(t)$. Some of them appear in the objective function - SMH and SOH - and the rest are variables which help to predict $\mathrm{SMH}, \mathrm{SOH}$ and/or are variables which are observed by farmers and processors. Certainly, there are an unlimited number of variables which contain the information needed. These variables may differ in degree of influence. We include variables which appear in other research and variables suggested by economic theory. The use of a different set of variables used is likely to give different results

The vector $Y$ contains:
soybean acreage planted
(A)
soybean crushed
soybean price, Decatur (PS)
soybean meal price (PM)
soybean oil price

Both $Y$ and $W$ have been adjusted for a linear trend. Note that all variables in $Y$ and $W$ are seasonally unadjusted series; therefore, seasonality patterns still remain. Seasonal dummy variables are added into the regression equations when the Granger test is performed. A pair of regressions is needed to test the causal hypothesis. They are:
i) Ho: $W \not+Y$ ( $W$ fails to Granger cause $Y$ ), or $B(k)=0^{4}$

Regression I: $Y(t)=\sum_{s=1}^{4} \alpha(s) Y(t-s)+\sum_{k=1}^{6} \beta(k) W(t-k)+C_{o}+V(t)$
where $Y_{0}$ is a constant term, and $V(t)$ is a white noise.
Accepting the null hypothesis means that $W$ does not "statis-
tically cause" Y.
ii) Ho: $\mathrm{Y} \not+\mathrm{W}$ or $\alpha(\mathrm{s})=0$.

4 Ignore seasonal terms for now. In order to test the null hypothesis Ho: all $\beta(k)=0$, the following regressions are run:
i) $Y(t)=\alpha_{01}+\sum_{s=1}^{4} \alpha(s) Y(t-s)+\varepsilon_{1 t}$
ii) $Y(t)=\alpha_{02}+\sum_{s=1}^{4} \alpha(s) Y(t-s)+\sum_{k=1}^{6} \beta(k) W(t-k)+\varepsilon_{2 t}$
where $\varepsilon_{1 t}$ and $\varepsilon_{2 t}$ are white noise residuals. An $F-s t a t i c$ can be computed from the sum of squares of errors from i) and ii). Let $\operatorname{SSE}_{1}$ and $\operatorname{SSE}_{2}$ be the sum of squares of errors from i) and ii), respectively, then

$$
\mathrm{F}_{\mathrm{c}}=\left(\frac{\mathrm{SSE}_{1}-\mathrm{SSE}_{2}}{\mathrm{q}}\right) /\left(\frac{\mathrm{SSE}_{2}}{\mathrm{~T}-\mathrm{p}-\mathrm{q}-1}\right)
$$

where $T$ is the number of observation and $p$ equals to four, $q$ equals to six in this case. $\mathrm{F}_{\mathrm{c}}$ is distributed as F with ( $\mathrm{q}, \mathrm{T}-\mathrm{p}-\mathrm{q}-1$ ) degrees of freedom.

$$
\begin{aligned}
\text { Regression II: } W(t)= & \sum_{k=1}^{4} \beta(k) W(t-k)+\sum_{s=1}^{6} \alpha(s) Y(t-s) \\
& +W_{0}+E(t)^{5}
\end{aligned}
$$

where $W_{0}$ is a constant term, and $E(t)$ is a white noise. Failure to reject the hypothesis implies that $Y$ does not "statistically cause" $W$.

In order to have $W$ appear on the right-hand side of decision rule $Y$ in the model, the acceptance of the second and rejection of the first hypotheses are needed. F-statistic is used for such a test. The residuals from each regression are also tested for white noise. Fisher's-Kappa statistic is used for a finite p periodogram ordinates.

The empirical part of the Granger-causality test requires the arrangement of data in vector $Y$ and $X$. The vector $Y$ is not necessarily the same as the original $Y(t)$ referring to choice variables and endogenous variables in the model. Vector $Y$ used in the empirical test refers to variables on the left-hand side, and vector X refers to variables on the right-hand side. The regression used is:
${ }^{5}$ Two regressions are needed for testing all $\alpha(s)=0$; they are:
i) $W(t)=\beta_{01}+\sum_{k=1}^{4} \beta(k) W(t-k)+\varepsilon_{1 t}$, and
ii) $W(t)=\beta_{02}+\sum_{k=1}^{4} \beta(k) W(t-k)+\sum_{s=1}^{6} \alpha(s) Y(t-s)+\varepsilon_{2 t}$
and the F-statistic is computed from the sum of squared errors from the two regressions.

$$
\begin{equation*}
Y(t)=\sum_{s=1}^{4} \alpha(s) Y(t-s)+\sum_{k=1}^{6} \beta(k) X(t-k)+c_{c}+e_{t} \tag{5.5}
\end{equation*}
$$

where $c_{0}$ is a constant, and $e_{t}$ is a white-noise disturbance term. All variables in $Y$ and $X$ are filtered with a 1inear trend. Table B. 4 in Appendix $B$ presents the results of the regressions in (5.5). Seasonal dummies are added into (5.5) when performing the regressions. The F-statistic reported in Table B. 4 is the statistic for testing the null hypothesis of all $\beta(k)$ in equation (5.5) equal to zero. We reject the null hypothesis for all $\beta(k)$ equal to zero if the F-statistic is greater than 2.29 or 3.18 at $\alpha$ being 0.05 and 0.01 , respectively. The summary of results of Table B .4 is in Table 4.2. The first two pairs of equations in Table $B .4$ are the results of the test whether the corn price received by farmers (CORPF) Granger-causes soybean acreage planted (A), or whether soybean acreage planted Granger-causes the corn price. For corn price to be used on the right-hand side of decision rule $A_{t}$, we must reject the hypothesis that all $\beta(k)=0$ in the first equation of $A$ and CORPF and fail to reject the hypothesis of all $\beta(k)=0$ in the second equation of CORPF and A. Rejecting the hypothesis in both equations results in two-way causality between the two variables. A one-way arrow in Table 5.2 represents unitary causality. Two-way arrows refer to bidirectional causality between the pair of variables.

Given variables in our finformation set, the following conclusion can be drawn:

Table 5.2. Summary of Granger-causality results

| Direction of causality | Y fails to Granger cause |
| :---: | :---: |
| A | CORPF |
| A |  |
| A |  |
| A |  |
| A | TB |
| SBHF |  |
| SBHF |  |
| SBHF |  |
| SBC | SBX |
| SBC |  |
| SBC |  |
| SC |  |
| SC |  |
| SC | - . |
| SC | SMH |
| SC | FIMP |
| PS |  |
| PS |  |
| PS | SOH |
| PS |  |
| PM |  |
| PM |  |
| PM | . |
| PO |  |
| PO |  |
| PO |  |
| PO |  |
| FIMP |  |

i) Corn price received by farmers and the T-bill interest rate can be used as exogenous variables on the right-hand side of the soybean acreage planted equation.
ii) Soybean exports are used on the right-hand side of decision rule $S B C$.
iii) Soybean meal stock and fish meal price can be used as exogenous variables on the right-hand side of the soybean crushed equation.
iv) Soybean oil stock Granger-causes soybean price.
v) Bidirectional causality, represented by the two-way arrows, suggests that further investigation is needed for soybean stocks, soybean meal price and soybean oil price. The highprotein animal units (HPAU) cannot be used as an exogenous variable in the model. The results from testing the pairs A-HPAU, SBHF-HPAU, and PM-HPAU, suggest that the soybean variables A, SBHF and PM can be used as exogenous variables in the livestock sector. However, the two-way causality result from SBC and HPAU may indicate further analysis is needed.

It is essential to keep in mind that the soybean model presented in this research does not incorporate other crops, such as corn. The Grangercausality tests which are appropriate depend upon the structure of the theoretical model.

It is worth mentioning that the Granger-causality test is also performed without seasonal dummies. Table 5.3 summarizes the results of such a test. The consistent results with Table 5.2 is soybean oil stock Granger-causes soybean price. The difference between Tables 5.2 and 5.3

Table 5.3. Summary of causality test

are due to seasonality patterns in our variables. All residuals from Table B. 4 in Appendix $B$ have been tested for white noise using the FisherKappa statistic. ${ }^{6}$ All residuals pass this test; however, Fisher-Kappa is not a powerful test for white noise. Granger-causality tests were performed on variables CORPF, FIMP, and COOP. The 1iterature on conventional econometric models of the soybean market indicate these variables are closely related with those in the soybean complex. Thus, they are added to the set of variables for the estimated model.

## Estimation

We mentioned in Chapter 4 that the set of equations (4.60), (4.61), (4.62), (4.63) and their cross-equation restrictions are to be used in estimation. One can use the maximum-1ikelihood procedure to get the estimates of the underlying parameters. To use this method, it must be possible to write the model in an explicit form such that the vector of residuals of all equations can be estimated. As Sargent (1978) stated,
"...optimizing rational expectations models does not entirely eliminate the need for side assumptions not grounded in economic theory. Some arbitrary assumptions about the nature of the serial-correlation structure of the disturbances and/or about strict econometric exogeneity are necessary in order to proceed with estimation."

This is exactly the case which cannot be avoided here.
Given the model,

$$
Y(t)=A(L) Y(t)+\beta(L) W(t)+v(t),
$$

$$
6_{\mathrm{F}-\mathrm{K} \text { statistic }}=\left(\frac{\mathrm{k}}{2(\mathrm{~m}-1)}\right) * \frac{\text { maximum periodogram }}{\text { sum of periodogram of } \mathrm{p} \text { ordinates }},
$$

where $k$ is the residuals degree of freedom in equation, and $m$ is half of number of observations.
where $v(t)$ is a normal vector with $E\left(v_{t} v_{t}^{\prime}\right)=V . \quad$ Let $\hat{V}=\frac{1}{T} \sum_{t=1}^{T} v_{t} v_{t}^{\prime}$ be the covariance matrix of $v(t)$. Thus, the value of the likelihood functions is as follows:

$$
\mathrm{L}(\theta)=-(1 / 2) \mathrm{mT} \log (2 \pi)-(1 / 2) \mathrm{T}\{\log |\mathrm{~V}|+\mathrm{m}\}
$$

where $m$ is the number of variates. For a smaller model maximizing $L(\theta)$, or minimizing $|\hat{V}|$ with respect to all free parameter vector $\theta$ is feasible. If $L_{u}$ is the value of the likelihood function of its unrestricted maximum (no restriction across equation), and $L_{r}$ is the value of the likelihood under restrictions of parameters underlying model, then $-2 \log _{e}\left(L_{r} / L_{u}\right)$ is asymptotically distributed as $x^{2}(q)$ where $q=q_{u}-q_{r}$ ( $q_{u}$ is the number of parameters to be estimated under unrestricted estimation, and $q_{r}$ is the number of parameters to be estimated under the restricted estimation). The restriction imposed by the model is rejected if the likelihood ratio is high (approximation of likelihood ratio is $T\left\{\log _{e}\left|\hat{V}_{r}\right|-\log _{e}\left|\hat{V}_{\mathbf{u}}\right|\right\}$ ). However, in a model such as this one, where there are fourteen variates and over twenty parameters to be discovered, jointly maximizing $L(\theta)$ over all the parameters is not practical. Hence, an iterating-search procedure is used.

The U.S. quarterly data from 1962 through 1977 is used with initial lagged values 1960 through 1961 being given. Detrending prior to the estimation is a device to isolate the indeterministic part from the deterministic parts of the series. The results from Fuller's and Grangercausality tests have indicated certain knowledge on the processes of all variables and to those which can be used to gain information.

Equation (4.60) can be written as:

$$
\begin{equation*}
A_{t+2}=a_{0}+a_{1} A_{t+1}+a_{2} P S_{t+2}+a_{3} P S_{t+1}^{7} \tag{5.6}
\end{equation*}
$$

From equation (4.44), $\mathrm{PS}_{\mathrm{t}+2}$ is:

$$
\begin{align*}
& P S_{t+2}=P 0+P 1 P S_{t+1}+P 2 P S_{t}+P 3 A_{t-4}+P 4 A_{t-8}+P 5 P_{t+2} \\
& + \text { P6 } P_{t+1}+P 7 P_{t}+P 8 P_{t-2}+P 9 C_{t+1}+P 10 P S C_{t} \\
& + \text { P11 }^{P} M_{t+1}+\text { P12 }^{P 0}{ }_{t+1}+\text { P13 SBX }_{t+2}+\text { P14 SBC }_{t+1} \\
& + \text { P15 SBC }_{\mathrm{t}}+\mathrm{P}^{2} \mathrm{SBC}_{\mathrm{t}-1}+\text { P17 SBHF }_{\mathrm{t}+1}+\text { P18 }^{\text {SBHF }_{\mathrm{t}}} \\
& + \text { P19 } \text { SBHF }_{\mathrm{t}-1}+\mathrm{P}^{20} \mathrm{SBHC}_{\mathrm{t}+2}+\mathrm{P}^{21} \mathrm{SBHC}_{\mathrm{t}+1}{ }^{8} \tag{5.7}
\end{align*}
$$

Given that

$$
\begin{align*}
& P_{t+2}=\beta P S_{t+2}+s_{t} ; \text { and }  \tag{4.6}\\
& (I-X(L)) S B X_{t+2}=e_{t+2}^{X} 9 \tag{5.8}
\end{align*}
$$

Equation (4.61) can be written as:

$$
\begin{equation*}
\mathrm{SBHF}_{\mathrm{t}+2}=\mathrm{F} 1 \mathrm{SBHF}_{\mathrm{t}+1}+\mathrm{F} 2 \mathrm{PS}_{\mathrm{t}+2}+\mathrm{F} 3 \mathrm{PS}_{\mathrm{t}+1}+\mathrm{F}^{2 \mathrm{PS}_{\mathrm{t}}}-\mathrm{F0} \tag{5.9}
\end{equation*}
$$

Equation (4.62) can be written as:

$$
\begin{align*}
S C_{t+2}= & G 1 S C_{t+1}+G 2 P M_{t+2}+G 3 P M_{t+1}+G 4 P O_{t+2} \\
& G 5 \mathrm{PO}_{t+1}+G 6 \mathrm{PS}_{t+2}-G 7 P S_{t+1} \tag{5.10}
\end{align*}
$$

Equation (4.63) can be written as:

$$
\begin{equation*}
\mathrm{SBC}_{\mathrm{t}+2}=\mathrm{H} 1 \mathrm{SBC}_{\mathrm{t}+1}+\mathrm{H} 2 \mathrm{PS}_{\mathrm{t}+2}+\mathrm{H} 3 \mathrm{PS}_{\mathrm{t}+1}+\mathrm{H} 4 \mathrm{PS}_{\mathrm{t}} \tag{5.11}
\end{equation*}
$$

The market clearing price for soybean meal is:

[^4]\[

$$
\begin{align*}
\mathrm{PM}_{\mathrm{t}+2}= & \mathrm{m}_{1} \mathrm{PM}_{\mathrm{t}+1}+\mathrm{m}_{2} \mathrm{PO}_{\mathrm{t}+1}+\mathrm{m}_{3} \mathrm{PS}_{\mathrm{t}+1}+\mathrm{m} 4 \mathrm{SC}_{\mathrm{t}+1} \\
& \mathrm{~m} 5 \mathrm{SC}_{\mathrm{t}}+\mathrm{m} 6 \mathrm{SMH}_{\mathrm{t}+1}+\mathrm{m} 7 \text { CORPF }_{\mathrm{t}+2}+\mathrm{m} 8 \mathrm{FIMP}_{\mathrm{t}+2} \tag{5.12}
\end{align*}
$$
\]

Where $\operatorname{CORPF}_{\mathrm{t}+2}$ and $\mathrm{FIMF}_{\mathrm{t}+2}$ have the following realized processes:

$$
\begin{align*}
& (I-F(L)) \operatorname{CORPF}_{t+2}=\ell_{t+2}^{R} ; \text { and }  \tag{5.13}\\
& (I-M(L)) \operatorname{FIMP}_{t+2}=\ell_{t+2}^{\mathrm{m}} ; \text { and } \tag{5.14}
\end{align*}
$$

The market clearing price of soybean oil is:

$$
\begin{align*}
\mathrm{PO}_{\mathrm{t}+2}= & +01 \mathrm{PO}_{\mathrm{t}+1}+02 \mathrm{PM}_{\mathrm{t}+1}+03 \mathrm{PS}_{\mathrm{t}+1}+04 \mathrm{SC}_{\mathrm{t}+1} \\
& +-5 \mathrm{SC}_{\mathrm{t}}+06 \mathrm{SOH}_{\mathrm{t}+1}+07 \mathrm{COOP}_{\mathrm{t}+2} \tag{5.15}
\end{align*}
$$

and

$$
\begin{equation*}
(I-O(L)) \operatorname{CoOP}_{t+2}=\ell_{t+2}^{0} ; \text { and } \tag{5.16}
\end{equation*}
$$

Equations (5.8), (5.13), (5.14) and (5.16) are realized stochastic process of SBX, CORPF, FIMP and COOP, respectively. These variables are contained in the agent's information set, and they help to predict the stochastic shifts in the endogenous variables. The crucial assumption concerning these four variables issthat their processes are known or discovered. Each $\ell_{t+2}$ term is assumed to be independent and identically normally distributed.

Equations (5.6), (5.7), (4.6), (5.8), (5.9), (5.10), (5.11), (5.12), (5.13), (5.14), (5.15), (5.16), (5.17) together with the SB, SM and SO equations and an identity of the soybean market, are estimated without restriction. The nonlinear estimation and simulation procedure in the SAS/ETS 79.6 version is used. Without any cross-equation restrictions on all parameters, they are the set of unrestricted models under the rationalexpectations hypothesis. This set of equations is called the Quasi-

Rational model is derived from the Granger tests. If the cross-equation restriction is correctly specified, the restricted version, RES model, will out-perform the Quasi-Rational model. However, this is not generally the case. The highly nonlinear restriction imposed by the RES model creates multi-collinearity among those free parameters which have to be discovered. Omitting certain variables may result in getting a wrong sign for certain parameters. To this date, no satisfactory procedure exists to account for this problem. The results of the Quasi-Rational model are represented in Table 5.4.1 and 5.4.2. Certain variables which fail the bi-variate Granger test are included in the Quasi-Rational model if they pass the multi-variate Granger test. Results in Table 5.4.1 are the OLS estimates of the Quasi-Rational model.

The sign of $P S(t)$ in the $A(t)$ equation is positive, as expected. The signs of PS ( $t$ ) in $\operatorname{SBHF}(t)$ and $\operatorname{SBC}(t)$ are negative, as suggested by theory. The sign of $P M(t), P O(t)$ and $P S(t)$ are all as expected. The additional information which does not appear in the structural model but is used in the model is FIMP, CORPF and COOP. These three variables are used as the result of the Granger-causality test.

Table 5.4.1. The unrestricted estimation of the Quasi-Rational Expectations Model ${ }^{\text {a }}$

|  | MSE | $\mathrm{R}^{2}$ | DW |
| :---: | :---: | :---: | :---: |
| $\mathrm{A}(\mathrm{t})=\underset{(0.01)^{\mathrm{b}}}{1.02 \mathrm{t}} \mathrm{~A}(\mathrm{t})+\underset{(0.41)}{3.34} \mathrm{PS}(\mathrm{t})-\underset{(0.58)}{2.89} \mathrm{PS}(\mathrm{t}-1)$ | 1.75 | 0.99 | 2.0 |
| $\operatorname{SBHF}(t)=\underset{(0.13)}{\underset{(0.39}{0.39} \operatorname{SBHF}(t-1)}-\underset{(8.9)}{25.83} \operatorname{PS}(t)+\underset{(12.14)}{18.55} \operatorname{PS}(t-1)+\underset{(9.7)}{13.56} \operatorname{PS}(t-2)$ | 2924.0 | 0.91 | 2.02 |
| $\begin{gathered} +389.16 \text { Fall }-\underset{(31.7)}{49.99} \text { Winter }-\underset{(14.6)}{110.8} \text { Spring } \\ (31.0) \end{gathered}$ |  |  |  |
| $\mathrm{SC}(\mathrm{t})=\underset{(0.1)}{\underset{(0.1)}{0.62} \mathrm{SC}(\mathrm{t}-1)+\underset{(0.12)}{0.35} \mathrm{PM}(\mathrm{t}) \underset{(0.14)}{-0.14} \mathrm{PM}(\mathrm{t}-1)+\underset{(0.63)}{3.12} \mathrm{PO}(\mathrm{t})}$ | 106.9 | 0.75 | 1.67 |
| $\underset{(0.86)}{-1.83} \operatorname{PO}(t-1)-\underset{(5.3)}{23.6} \operatorname{PS}(t)+\underset{(5.47)}{8.85} \operatorname{PS}(t-1)+\underset{(2.7)}{1.03} \text { Fa11 }$ |  |  |  |
| $+\underset{(2.6)}{9.47} \text { Winter }+\underset{(2.3)}{2.31} \text { Spring }$ |  |  |  |
|  | 4435.0 | 0.90 | 2.04 |
| $+\underset{(38.7)}{305.01} \text { Fall }+\underset{(38.2)}{12.84} \text { Winter }-\underset{(20.17)}{100.6} \text { Spring }$ |  |  |  |

[^5]Table 5.4.1 - continued


Table 5.4.1 - continued

|  | MSE | $\mathrm{R}^{2}$ | DW |
| :---: | :---: | :---: | :---: |
| $\mathrm{PO}(t)=\underset{(0.26)}{0.44} \mathrm{PO}(t-1)+\underset{(0.04)}{0.047} \mathrm{PM}(t-1)-\underset{(2.0)}{-1.48} \mathrm{PS}(t-1)+\underset{(0.042)}{0.004} \mathrm{SC}(t-1)$ | 7.74 | 0.85 | 2.2 |
|  |  |  |  |
| $\begin{aligned} & +\underset{(0.15)}{0.26} \operatorname{COOP}(t-2)-0.22 \operatorname{COOP}(t-3) \\ & (0.13) \end{aligned}$ |  |  |  |

The h-statistic ${ }^{10}$ of $S C(t)$ is 1.72 which means that we reject the zero autocorrelation at the five percent level of significant. Therefore, the first-order autocorrelation model is imposed to $S C(t)$. Table 5.4.2 is the result of the unrestricted estimation of the model. The model is run under seemingly unrelated regression. The results of the estimation are reported in Table 5.4.2. Using the likelihood ratio test the model in Table 5.4.1 is tested against the model in Table 5.4.2. We reject the model in Table 5.4.1. Thus, there exists serial correlation in SC.

Restricted estimation under the rational expectation hypothesis
There are nine underlying parameters to be discovered. They are g0, $\mathrm{g} 1, \mathrm{~g} 2, \mathrm{~g} 3, \mathrm{~d} 1, \mathrm{~d} 2, \mathrm{~d} 3, \mathrm{~d} 4$, and $\beta$. We assume that the discount factors " $b$ " and " $\delta$ " are 0.99 and 0.9 , respectively. We also assume that soybean meal yield and soybean oil yield (somsc and soosc) are 0.487 and 0.108 , respectively, and the adjustment factor for soybean yield ( $y$ ) is $0.98 * y i e l d$. The discount factior " $\delta$ " is assumed to be lower than " $b$ ", implying that soybean processors have a higher rate of return than farmers. Whether these assumptions are realistic is left for further research.

$$
\begin{gathered}
10 \text { Let } r=1-0.5 * \mathrm{DW}, \mathrm{n}=\text { number of observations } \\
\operatorname{Var}(\mathrm{bl})=\text { variance of lagged endogenous variable, then } \\
h=r(n / 1-n \operatorname{Var}(\mathrm{bl}))^{1 / 2}
\end{gathered}
$$

Table 5.4.2. The unrestricted estimation of Quasi-Rational Expectations model - using seemingly unrelated regressions

|  |  | MSE | $\mathrm{R}^{2}$ | DW |
| :---: | :---: | :---: | :---: | :---: |
| $A(t)=$ | $\underset{(0.01)}{1.02} \mathrm{~A}(\mathrm{t}-4)+\underset{(0.39)}{3.36} \mathrm{PS}(\mathrm{t})-\underset{(0.55)}{2.88} \mathrm{PS}(\mathrm{t}-1)$ | 1.75 | 0.99 | 2.0 |
| $\operatorname{SBHF}(\mathrm{t})=$ | $\begin{gathered} \underset{(0.11)}{0.32} \operatorname{SBHF}(t-1)-\underset{(8.7)}{27.17} \operatorname{PS}(t) \end{gathered}$ | 2956.0 | 0.91 | 1.86 |
|  | $+\underset{(11.7)}{15.02} \operatorname{PS}(t-1)+\underset{(9.1)}{16.6} \operatorname{PS}(t-2)$ |  |  |  |
|  | $+\underset{(23.3)}{278.69} \text { Fall }+\underset{(28.3)}{32.99} \text { Winter }-\underset{(13.8)}{110.6} \text { Spring }$ |  |  |  |
| $\mathrm{SC}(\mathrm{t})=$ | $\underset{(0.14)}{0.08} \mathrm{SC}(\mathrm{t}-1)+\underset{(0.09)}{0.32} \mathrm{PM}(\mathrm{t})+\underset{(0.14)}{0.11} \mathrm{PM}(\mathrm{t}-1)$ | 98.4 | 0.78 | 2.13 |
|  | $+\underset{(0.62)}{2.23} \mathrm{PO}(\mathrm{t})+\underset{(0.7)}{0.24} \mathrm{PO}(\mathrm{t}-1)-\underset{(4.4)}{21.2} \mathrm{PS}(\mathrm{t})$ |  |  |  |
|  | $\underset{(5.2)}{-9.12} \operatorname{PS}(t-1)-\underset{(2.1)}{2.5} \text { Fall }+\underset{(1.9)}{5.8} \text { Winter }$ |  |  |  |
|  | $+\underset{(1.9)}{5.04} \text { Spring; where } \hat{\rho}=0.70$ |  |  |  |
| $\operatorname{SBC}(\mathrm{t})=$ | $\begin{array}{cc} 0.15 \\ (0.13) & \operatorname{SBC}(t-1) \\ (10.8) \end{array}$ | 4518.0 | 0.89 | 1.75 |
|  | $-14.4 \mathrm{PS}(\mathrm{t}-1)+7.45 \mathrm{PS}(\mathrm{t}-2)+\underset{(10.7)}{275.65 \mathrm{Fall}}$ |  |  |  |
|  | $+\underset{(34.8)}{39.18} \text { Winter }-\underset{(18.9)}{86.5} \text { Spring }$ |  |  |  |

As mentioned earlier, the superior procedure to solve for the free parameters - nine underlying parameters and those which are characterized by the realization of exogenous variables (coefficients of autoregressive models of exogenous variables), is to solve the nonlinear system of thirteen equations with cross-equation restrictions simultaneously (assuming that SBHC is given fixed).

This procedure, however, is very expensive. Thus, the estimation is constrainted by the computer cost. Nonlinear joint generalized least squares are run on the model. Using PROC SYSNLIN in SAS/ETS 79.6 version, the underlying parameters can be discovered. The problems concerning the estimation are that the system may not be easy to converge and it generally takes time to solve the system. If the system is successfully solved, all necessary statistics can be obtained from the PROC SYSNLIN. Simulation can also be done simultaneously with estimation by using PROC SIMNLIN, providing that there are no missing data. The estimates of all underlying parameters are reported in Table 5.5.1. Some negative values may be due to omitted variables - and misspecify the model.

The adjustment cost parameters - d1, d4, g1, and g3, are all positive as expected. The higher the values of these variables, the slower the speed of adjustment of soybean acreage planted, soybean Inventory on-farm, soybean crushed and soybean inventory off-farm. The values of lagged choice variables depend upon these adjustment costs (as shown in Chapter four).

Table 5.5.1. Estimates of parameters of the soybean model (RES model) ${ }^{\text {a }}$

$$
\begin{aligned}
& G 0=0.4558 \\
& D 1=8.12 \\
& \text { (18.9) } \\
& G 1=16.75 \\
& \mathrm{D} 2=2972.89 \\
& \text { (7920) } \\
& G 2=-1109.2 \\
& \mathrm{D} 3=-0.0056 \\
& \text { (3259) } \\
& \text { (0.03) } \\
& G 3=0.0007 \\
& D 4=110.01 \\
& \text { (0.0004)** } \\
& \text { (273) } \\
& \beta=0.96 \\
& \text { (0.01)* }
\end{aligned}
$$

The $\log _{e}$ of the determinant of variance-covariance matrix is 13.95 .
$a_{\text {The following parameters are assumed priori: }}$
$\begin{array}{ll}\mathrm{b}=0.99 & \text { somsc }=0.487 \\ \delta=0.9 & \text { soosc }=0.108 .\end{array}$
*Significant at $\alpha=0.05$.
${ }^{* *}$ Significant at $\alpha=0.10$.

The negative values of $G_{2}$ and $D_{3}$ may be due to misspecification in the model. Using the approximated value of likelihood ratio $T\left[\log _{e}\left|V_{r}\right|-\log _{e}\left|V_{u}\right|\right]$, the value of likelihood ratio is 222.5 , which is very high. Thus, we reject the restricted model. Given the results of the test, one should go back to the structural model and reformulate it. Certain improvements can be made through incorporating the other crop such as corn. Other inputs such as fertilizers and machinery may be taken into consideration. Of course, these revisions will add more complication to the model. Due to high computation costs, further implications under the RES model will not be performed. Only the Quasi-Rational model is used for dynamic simulation in Chapter six.

Given the results in Table 5.5.1 and Table 5.4.2, the structural decision rules can be written as in Table 5.5.2. The assumptions made the farmers' discount factor being 0.99, the processors' discount factor being 0.9 , and the soybean adjustment factor yield being 0.98 *yield. The higher the soybean yield per harvested acre, the higher the coefficient of the expected future prices. The higher the yield, the higher the profit opportunities for farmers. The coefficient has the inverse relationship with the underlying parameter $D_{1}$. The higher the parameter $D_{1}$ (higher adjustment cost), the lower the coefficient of the future price and lower the speed of adjustment of soybean acreage planted.

The coefficient of the expected price change in the soybean inventory on-farm ( $\mathrm{SBHF}_{\mathrm{t}}$ ) depends upon the parameter D4 (the adjustment cost parameter). The higher the parameter D4, the lower the price coefficient and the lower the speed of adjustment of the inventory on-farm.

Table 5.5.2. The structural decision rules of the soybean market

$$
\begin{aligned}
& A_{t}=\quad A_{t-4}+(0.1182 * \text { yield }) \sum_{i=0}^{\infty}(0.99)^{4 i} 0.96 E\left(P S_{t+2+4 i}\right) \\
& \mathrm{SBHF}_{\mathrm{t}}=0.382 \mathrm{SBHF}_{\mathrm{t}-1}+(0.0035) \sum_{i=0}^{\infty}(0.378)^{i}\left[0.99 \mathrm{E}\left(\mathrm{PS}_{t+i+1}\right)-\mathrm{E}\left(\mathrm{PS}_{\mathrm{t}+\mathrm{i}}\right)\right] \\
& S C_{t}=0.63 S C_{t-1}+0.037 * \frac{\operatorname{somsc}}{20} * \sum_{i=0}^{\infty}(0.563)^{i} E\left(P_{t+i}\right)+0.037 * \operatorname{soosc} \sum_{i=0}^{\infty}(0.563)^{i} E\left(P_{t+i}\right) \\
& -0.037 \sum_{i=0}^{\infty}(0.563)^{i} \mathrm{E}\left(\mathrm{PS}{ }_{\mathrm{t}+\mathrm{i}}\right) \\
& S B C_{t}=0.021 \quad S C_{t-1}+44.28 \sum_{i=0}^{\infty}(0.0189)^{i}\left[0.9 E\left(P S_{t+i+1}\right)-E\left(P S_{t+i}\right)\right]
\end{aligned}
$$

The coefficient " 0.037 " in the soybean crushed also depends upon the adjustment cost parameter $g_{1}$. The higher the adjustment cost parameter, the lower the coefficient and the lower the speed of adjustment of the soybean crushed.

The interpretation on the coefficient of the price change in the soybean inventory off-farm $\left(S B C_{t}\right)$ is the same as $S B H F_{t}$. The negative values of $G_{2}$ and $D_{3}$ may imply "reward" concerning crushing, holding inventory on-farm and off-farm; however, this is difficult to interpret.

## Estimation of adaptive expectation hypothesis

The infinite lagged length of $P S, P M$ and $P O$ can be replaced with finite lagged length by using the Koyck transformation.

Table 5.6 is the estimated equations of the Adaptive Price Expectations model. The lagged prices are proxies of anticipatory future prices. Seasonal dummy variables are added to the equations in order to take care of seasonality in time series. No additional information is used. All coefficients are estimated without any restrictions as in the rational-expectation version. The average coefficients of lagged price PS in soybean acreage equation ( $A_{t}$ ) are positive. The positive lagged price in this acreage response implies the cobweb phenomenon in supply function. The sign of the current price of soybeans in both stock equations are negative as expected. The sign of the lagged value of the soybean price should be positive; however, this is not the case of the off-farm inventory equation. The signs in the crushing equation are not as expected.

Table 5.6. Estimation equations under Adaptive Expectations Hypothesis ${ }^{\text {a }}$

|  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

${ }^{a}$ A11 variables in soybean acreage equation are multiplied by quarterly dummy variable -- one in planting quarter and zero otherwise.
${ }^{b}$ Standard deviation.

The Figures 5.5 through 5.8 illustrate the simulation result of the Adaptive model over the sample period.

## Estimation of Cash-Futures Price Expectations Model

There are certain clarifications concerning data in the futures market which have to be pointed out. As mentioned earlier, all data in this research are based upon the crop year of soybeans, soybean meal and soybean oil (September through August for soybean and October through September for soybean meal and oil). The rearrangement also has to be done for the futures-prices data. All futures-prices data are reported according to trading years, which start from different months according to monthly delivery. The November contract (harvested contract) for soybean and December contract for soybean meal and oil are used here.

The quarterly average futures prices of soybeans are computed from the high and low monthly average of the quarter. For example, the soybean average futures prices for the first quarter of crop year 1970 (September, October and November 1970) are the high-low monthly average of soybean November 1970 contract, and the soybean average futures prices for the second quarter (December 1970 - January and February 1971) are the highlow monthly average of soybean November 1971 contract. Although the processes of the futures prices of beans, meal and oil are not assumed, we compute the autoregressive model of PSF, POF and PMF in Table B. 5 in Appendix B. Comparing the mean square errors of the same processes in Table 5.1 and B.5, it is seen that cash prices of beans, meal and oil have higher mean square errors. We realize that the cash and futures prices are created from different stochastic processes; comparing the mean square


Figure 5.5. Estimated and actual values of soybean acreage planted


Figure 5.6. Estimated and actual values of soybean crushed


Figure 5.7. Estimated and actual values of soybean inventory on-farr.

errors by no means implies anything about the process of the two price categories.

Using Koyck transformation on the one period forward on $A_{t}, \operatorname{SBHF}_{t}$, , $S C_{t}$ and $S B C_{t}$, we can eliminate the infinite sum in equations (4.70) through (4.73) and obtain the operational version of (4.70) through (4.73). Table 5.7 presents the results of the OLS estimates of the cashfutures price model.

The positive sign on $\mathrm{PSF}_{\mathrm{t}}$ in the soybean acreage function implies that farmers will grow more beans if they expect a higher price at harvest time. This result is what we expect. The positive sign of $A_{t-4}$ is expected. The positive sign of $A_{t-4}$ reflects the positive value of parameter $D_{1}$ in the rational-expectations model, which implies the speed of adjustment of soybean acreage planted. Although there is no a priori belief in the adaptive and cash-futures models for the sign of $A_{t-4}$, the positive result is matched with our theory and farmers' practices.

The lag of soybean inventory on-farm ( SBHF $_{t}$ ) has a positive sign in all three models. This is what we expect. The sign of SBHF ${ }_{t-1}$ in the RES model depends upon the sign of the parameter $D_{4}$, which is also positive.

The sign of $\mathrm{PSF}_{\mathrm{t}+2, \mathrm{t}}$ (soybean futures price quotes at t for $\mathrm{t}+2$ contract - harvesting period) is positive as we expect. The higher the harvested price, the more likely farmers will plant soybeans. One also expects the sign of $P S_{t+2, t}$ and $P S F_{t+2, t-1}$ to be positive in the off-farm inventory equation. The signs of all prices in crushing and off-farm inventory are not as expected.

Table 5.7. Estimates of cash-futures price expectations model ${ }^{\text {a }}$

| $A_{t}$ : | $A_{t-4}$ | $A_{t-8}$ |  |  | $\mathrm{PSF}_{\mathrm{t}+2, \mathrm{t}}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} 0.7758 \\ (0.12)^{\mathrm{b}} \end{gathered}$ |  | $\begin{gathered} 0.28 \\ (0.13) \end{gathered}$ |  | $\begin{gathered} 0.024 \\ (0.05) \end{gathered}$ |  |
| $\mathrm{SBHF}_{\mathrm{t}}$ : | $\mathrm{SBHF}_{\mathrm{t}-1}$ |  | SBHF $_{\text {t-2 }}$ |  | $\mathrm{PSF}_{\mathrm{t}+2, \mathrm{t}}$ |  |
|  | $\begin{gathered} 0.32 \\ (0.12) \end{gathered}$ |  | $\begin{aligned} & -0.17 \\ & (0.12) \end{aligned}$ |  | $\begin{gathered} -5.42 \\ (1.9) \end{gathered}$ |  |
| SC ${ }_{t}$ : | $S_{\text {t-1 }}$ | SC $\mathrm{t}^{\text {-2 }}$ | $\mathrm{PSF}_{t+2, t-1}$ |  | $\mathrm{POF}_{t+2, t-1}$ |  |
|  | $\begin{gathered} 0.73 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.25 \\ & (0.13) \end{aligned}$ | $\begin{gathered} 1.13 \\ (13.7) \end{gathered}$ |  | $\begin{gathered} -0.84 \\ (1.6) \end{gathered}$ |  |
| $S B C_{t}$ : | $\mathrm{SBC}_{\mathrm{t}-1}$ | $\mathrm{SBC}_{\text {t-2 }}$ | $\mathrm{PSF}_{\mathrm{t}+2, \mathrm{t}}$ | $\mathrm{PSF}_{\mathrm{t}+2, \mathrm{t}-1}$ |  | Fall |
|  | $\begin{gathered} 0.14 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.05 \\ & (0.13) \end{aligned}$ | $\begin{aligned} & -3.44 \\ & (2.04) \end{aligned}$ | ${ }_{(9.3)}^{-17.6}$ |  | $\begin{aligned} & 252.11 \\ & (31.6) \end{aligned}$ |

${ }^{a_{\text {All }}}$ variables are filtered by linear trend, and, as usual, all variables in soybean acreage planted equations are multiplied by quarterly dummy variable -- having value one in planting quarter and zero otherwise.
$\mathrm{b}_{\text {Standard }}$ errors.

| PSF $_{\text {t }+2, \mathrm{t}-1}$ | Fall | Winter | Spring |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 13.49 |  | 246.4 | -51.5 | -71.6 | 3599.0 | 0.87 | 1.92 |
| $(7.9)$ | $(24.1)$ | $(40.3)$ | $(28.9)$ |  |  |  |  |
|  |  | Fall | Winter | Spring |  |  |  |
| PMF $_{\text {t+2,t-1 }}$ |  | 6.14 | 8.41 | -2.69 | 157.7 | 0.58 | 1.92 |
| -0.06 |  | $(3.2)$ | $(3.01)$ | $(3.02)$ |  |  |  |
| $(0.32)$ |  |  |  |  |  |  |  |
| Winter | Spring |  |  |  | 4451.0 | 0.88 | 1.90 |
| 35.53 | -77.27 |  |  |  |  |  |  |

The predicted values of $A_{t}, S B H F_{t}, S C_{t}$ and $S B C_{t}$ from Table 5.7 are not promising. Had we applied the same technique as in the rational expectations hypothesis to the cash-futures prices model, we certainly will improve the model. This is left for further research.

Concerning the implication of the model in Chapter six, only the Quasi-Rational model will be used for a dynamic simulation.

## CHAPTER 6. IMPLICATIONS AND CONCLUSIONS

The theoretical derivation and estimation of the soybean model under the three price-expectation regimes have been presented in Chapters four and five. The rational-expectations model implies that agents who have a correct perception of market behavior will out-perform those who do not. There are, however, drawbacks encountered in this research.

First, gathering and processing information is costly. For example, the estimation of the restricted model (RES) is more expensive than the other models. The cost of estimating such a model increases proportionally with the complexity of the model. Second, if there is a structural change in the economy, it may take a number of quarters for the model to converge to "rational-expectations equilibrium". In one lifetime, we may never see the model converge to such an equilibrium. Although we cannot deny that, one can gain a great deal from the model.

Some suggestions for further research are:
(i) incorporating corn as a competing crop, and
(ii) modeling the demand for soybean meal and soybean oil. Certainly, this suggestion will add more complications to the model. To solve the model such as suggested, the restricted estimation procedure is not practical. A quasi-rational-expectations version is practical, however.

Given the structural decision rules in Table 5.5.2, the elasticities of the four decision variables at "time $t$ " with respect to the expected price of soybeans $E\left(\mathrm{PS}_{\mathrm{t}+\mathrm{k}}\right)$ can be obtained as the following:
(i) Elasticity of soybean acreage planted at $t$ with respect to the expected price of soybean $E\left(\mathrm{PS}_{t+k}\right)$,

$$
\begin{aligned}
& \varepsilon\left(\mathrm{A}(\mathrm{t}), \mathrm{E}\left(\mathrm{PS}_{\mathrm{t}+2}\right)\right)=0.24 \\
& \text { yield }=30, \text { PS }=5, \text { acre }=70
\end{aligned}
$$

(ii) Elasticity of soybean inventory on-farm at $t$ with respect to the change in soybean price at $t+1$.

$$
\varepsilon\left(\operatorname{SBHF}(t), \Delta \mathrm{PS}_{t+1}\right)=0.00002
$$

(iii) Elasticity of soybean crush at time $t$ with respect to the expected price of soybean meal, oil and soybeans,

$$
\begin{aligned}
& \varepsilon\left(S C(t), E\left(P M_{t}\right)\right)=0.0006 \\
& \varepsilon\left(S C(t), E\left(P O_{t}\right)\right)=0.0004 \\
& \varepsilon\left(S C(t), E\left(P S_{t}\right)\right)=-0.001
\end{aligned}
$$

(iv) Elasticity of soybean Inventory off-farm at $t$ with respect to the expected price of soybeans at $\mathrm{t}+1$

$$
\begin{aligned}
\varepsilon\left(\operatorname{SBC}(t), E\left(\mathrm{PS}_{t+1}\right)\right) & =0.67, \text { and } \\
\varepsilon(\operatorname{SBC}(t), \operatorname{PS}(t)) & =-0.68
\end{aligned}
$$

All these elasticities depend upon the underlying parameters, such as adjustment cost parameters, discount factors, and adjustment factors for yield. We expect that the higher the adjustment factor for yield, the higher the elasticity $\varepsilon\left(A(t), E\left(P S_{t+k}\right)\right.$ ), and the higher the adjustment cost, the lower the elasticities for the four decision variables.

Given the Quasi-Rational model, a dynamic simulation is performed over the sampling period. Using the structural relationships in Chapter four, some variables, such as soybean price received by farmers ( $P_{t}$ ),
soybean production ( $\mathrm{SB}_{\mathrm{t}}$ ), soybean meal production ( $\mathrm{SM}_{\mathrm{t}}$ ) and soybean ofl production ( $\mathrm{SO}_{\mathrm{t}}$ ) can be computed.

Figure 6.1 is the result of the predicted values of the soybean price (PS) from the dynamic simulation of the Quasi-Rational model. The predicted values are plotted against time (the first quarter of the crop year 1968 through the last quarter of the crop year 1977). The statistics which we use to measure the simulation performance are the percentage root mean-square error (\% RMSE) and the turning point error (TPE). The \% RMSE for PS is 0.02 , and TPE is 0.04 . These figures indicate good performance for PS. Unfortunately, we are not able to get good results of the simulation on the restrictive model. The rejection of the restrictive model certainly indicates that we may have wrong cross-equation restrictions. Further research is needed to improve the formulation and estimation of the model and to incorporate corn and other important variables in the model.

There is also a problem of detrending, which is used in the estimation procedures that may affect the result of the simulation. The OLS estimate of trend is static and deterministic. The time series used in this research may "drift" downward and/or upward over time. If they are drifting, they possess stochastic properties which must be expressed as the outcome of a process operating through time. It is very difficult, in fact, to tell whether time series possess deterministic trend or stachastic drift. If we model deterministically, when it is stochastic, we may make serious errors in the prediction of future values. To deal with stochastic drift certainly is a difficult task.


Figure 6.1. Dynamic simulation result of soybean price from Quasi-Rational model

The purpose of undertaking this research is to strengthen the relationships between economic theory of expectations and econometric practice. The economic time series interpretation approach in this research is the new breed in modelling econometric models. Although there is room to improve the results obtained from this research, the attempt is accomplished for the present purpose.

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APPENDIX A. DERIVATION OF OPTIMAL DECISION RULES

$$
\begin{align*}
\operatorname{Max}_{\left\{n_{t}\right\}}: & \lim _{\rightarrow \infty} E_{t} \sum_{j=0}^{N} \beta^{j}\left[\left(\gamma_{0}+a_{t+j}-w_{t+j}\right) n_{t+j}-\left(\frac{\gamma_{1}}{2}\right) n_{t+j}^{2}\right. \\
& \left.-(\delta / 2)\left(n_{t+j}-n_{t+j-1}\right)^{2}\right] \tag{1.}
\end{align*}
$$

Differentiating (1) with respect to $n_{t+j}, j=0,1,2 \ldots N-1$, and setting each derivative equal to zero gives the system of stochastic Euler equations

$$
\begin{gather*}
\beta E_{t+j} n_{t+j+1}+\phi n_{t+j}+n_{t+j-1}=\left(\frac{1}{\delta}\right)\left(w_{t+j}-a_{t+j}-\gamma_{0}\right) \\
j=0,1,2 \ldots N-1 \tag{2}
\end{gather*}
$$

where $\phi=-\left(\left(\gamma_{1} / \delta\right)+1+\beta\right)$
The transversality condition is

$$
\begin{equation*}
\lim _{N \rightarrow \infty} E_{t} \beta^{N}\left[\gamma_{0}+a_{t+N}-w_{t+N}-\gamma_{1} n_{t+N}-\delta\left(n_{t+N}-n_{t+N-1}\right)\right]=0 \tag{3}
\end{equation*}
$$

Using the backward operator "L", the characteristic polynomial of (2) is:

$$
\left(1+\left(\frac{\phi}{\beta}\right) L+\left(\frac{1}{\beta}\right) L^{2}\right)=\left(1-\rho_{1} L\right)\left(1-\rho_{2} L\right)
$$

where $0<\rho_{1}<1$ and $\rho_{2}=\frac{1}{\beta \rho_{1}}$

$$
\begin{equation*}
\text { Thus, }\left(1-\rho_{1} L\right)\left(1-\rho_{2} L\right) n_{t}=\frac{1}{\beta \delta} E_{t}\left(w_{t-1}-a_{t-1}-\gamma_{0}\right) \tag{4}
\end{equation*}
$$

or

$$
\begin{aligned}
\left(1-\rho_{1} L\right) n_{t} & =(\beta \delta)^{-1} \frac{1}{1-\rho_{2} L} E_{t}\left(w_{t-1}-a_{t-1}-\gamma_{0}\right) \\
& =-(\beta \delta)^{-1} \frac{\left(\rho_{2} L\right)^{-1}}{1-\left(\rho_{2} L\right)^{-1}} E_{t}\left(w_{t-1}-a_{t-1}-\gamma_{0}\right)
\end{aligned}
$$

$$
\begin{aligned}
\left(1-\rho_{1} L\right) n_{t} & =-\frac{1}{\beta \delta}\left(\beta \rho_{1}\right) \frac{L^{-1}}{1-\left(\rho_{2} L\right)^{-1}} E_{t}\left(w_{t-1}-a_{t-1}-\gamma_{0}\right) \\
& =-\left(\frac{\rho_{1}}{\delta}\right) \frac{1}{1-\left(\rho_{2} L\right)^{-1}} E_{t}\left(w_{t}-a_{t}-\gamma_{0}\right)
\end{aligned}
$$

Thus, the unique solution of the Euler equations that satisfied the transversality condition is

$$
\begin{equation*}
n_{t}=\rho_{1} n_{t-1}-\left(\frac{\rho_{1}}{\delta}\right) \sum_{j=0}^{\infty}(\lambda)^{j_{E}}{ }_{t}\left(w_{t+j}-a_{t+j}-\gamma_{0}\right) ; \lambda=\frac{1}{\rho_{2}} \tag{5}
\end{equation*}
$$

To get the decision rule for $n_{t}$ in an explicit form, we need to assume the representative of $a_{t}$ and $w_{t}$.

$$
\text { Let } U=(1,0,0, \ldots) \text { of order (1xp) }
$$

$$
\begin{align*}
& a_{t}=\alpha_{1} a_{t-1}+\ldots \alpha_{q} a_{t-q}+v_{t}^{a} \text { or } \\
& a_{t}-\alpha_{1} a_{t-1}-\alpha_{q} a_{t-q}=v_{t}^{a} \text { or } \\
&\left(1-\alpha_{1} L-\alpha_{2}^{L} L^{2}-\alpha_{q} L^{q}\right) a_{t}=v_{t}^{a} \text { or }  \tag{7}\\
& \alpha(1) a_{t}=v_{t}^{a} \text { or }  \tag{8}\\
& a_{t}=\alpha(L)^{-1} v_{t}^{a} \tag{9}
\end{align*}
$$

Note that equation (8) and (9) imply the invertibility of $a_{t}$.
In summary, we can write $a_{t}$ as:

$$
\begin{equation*}
a_{t}=\alpha(L)^{-1} v_{t}^{a}=\psi(L) v_{t}^{a}=\left[\sum_{j=0}^{\infty} \psi_{j} L^{j}\right] v_{t}^{a} \tag{10}
\end{equation*}
$$

Let $w_{t}$ be the first element of a vector $X_{t}$, which has the following process:
or

$$
\begin{gather*}
x_{t}-\varepsilon_{1} x_{t-1}-\varepsilon_{2} x_{t-2}-\varepsilon_{r} x_{t-r}=v_{t}^{x}  \tag{11}\\
\left(1-\varepsilon_{1} L-\varepsilon_{2} L^{2}-\varepsilon_{r} L^{r}\right) x_{t}=v_{t}^{x}  \tag{12}\\
\varepsilon(L) x_{t}=v_{t}^{x}=G(L)^{-1} x_{t} \tag{13}
\end{gather*}
$$

Thus,

$$
\begin{equation*}
x_{t}=\varepsilon(L)^{-1} v_{t}=G(L) v_{t}^{x}=\left[\sum_{j=0}^{\infty} G_{j} L^{j}\right] v_{t}^{x} \tag{14}
\end{equation*}
$$

From (5) we can write it as:
where $\gamma_{0}$ is assumed to equal zero.

Note that

$$
\begin{aligned}
& E_{t} X_{t+k}=\left[\sum_{j=k}^{\infty} G_{j} L^{j-k}\right] v_{t}^{x} \\
& \sum_{k=0}^{\infty}\left(\lambda^{k}\right) E_{t} X_{t+k}=\sum_{k=0}^{\infty}\left(\lambda^{k}\right)\left[\sum_{j=k}^{\infty} G_{j} L^{j-k}\right] v_{t}^{x} \\
& =\Phi(\mathrm{L}) \mathrm{v}_{\mathrm{t}}^{\mathrm{X}} \text { where } \\
& \Phi(L)=\sum_{k=0}^{\infty} \sum_{j=k}^{\infty} \lambda^{k}{ }_{G}{ }_{j}{ }^{j} L^{-k} \\
& =\sum_{j=0}^{\infty} \sum_{k=0}^{j} \lambda^{k_{G}}{ }_{j}{ }^{j} L_{L}^{-k} \\
& =\sum_{j=0}^{\infty} G_{j} L^{j} \sum_{k=0}^{j} \lambda^{k} L^{-k} \\
& \left.=\sum_{j=0}^{\infty} G_{j} L^{j}\left[1-\lambda^{j+1} L^{-j-1}\right) /(]-\lambda L^{-1}\right] \\
& =\left(\sum_{j=0}^{\infty} G_{j} L^{j}-\lambda L^{-1} \sum_{j=0}^{\infty} G_{j} \lambda^{j}\right) /\left(1-\lambda L^{-1}\right) \\
& =\left(G(L)-\lambda L^{-1} G(\lambda)\right) /\left(1-\lambda L^{-1}\right)
\end{aligned}
$$

Thus,

Thus,

$$
\begin{align*}
\sum_{k=0}^{\infty} \lambda^{k} E_{t} X_{t+k} & =\left[\left(G(L)-\lambda L^{-1} G(\lambda)\right) /\left(1-\lambda L^{-1}\right)\right] v_{t}^{x} \\
& =\left[\left(G(L)-\lambda L^{-1} G(\lambda)\right) /\left(1-\lambda L^{-1}\right)\right] G(L)^{-1} x_{t} \\
& =\left[\left(I-L^{-1} \lambda G(\lambda) G(L)^{-1}\right) /\left(1-\lambda L^{-1}\right)\right] x_{t} \\
& =\left[\left(I-L^{-1} \lambda \varepsilon(\lambda)^{-1} \varepsilon(L)\right) /\left(1-\lambda L^{-1}\right)\right] x_{t} \tag{16}
\end{align*}
$$

Let $x_{t}$ have $r^{\text {th }}$ order of autoregressive representative as in (12), thus:

$$
\begin{aligned}
\varepsilon(L) /\left(1-\lambda L^{-1}\right)= & {\left[-\varepsilon_{r^{L}} L^{r}-\varepsilon_{r-1} L^{r-1}-\ldots \varepsilon L+I\right] /\left(1-\lambda L^{-1}\right) } \\
= & -\varepsilon_{r^{1}} L^{r}-\left(\varepsilon_{r-1}+\lambda \varepsilon_{r}\right) L^{r-1}-\ldots \\
& +\left(I-\lambda \varepsilon_{1}-\lambda^{r} \varepsilon_{r}\right) /\left(1-\lambda L^{-1}\right)
\end{aligned}
$$

Thus, $\quad L^{-1} \lambda_{\varepsilon}(\lambda)^{-1} \varepsilon(L) /\left(1-\lambda L^{-1}\right)=L^{-1} \lambda \varepsilon(\lambda)^{-1}\left[-\varepsilon_{r} L^{r}{ }^{r}\left(\varepsilon_{r-1}+\lambda \varepsilon_{r}\right) L^{r-1}-\ldots\right]$

$$
+L^{-1} \lambda I /\left(1-\lambda L^{-1}\right)
$$

Thus, $\left(I-L^{-1} \lambda \varepsilon\left(\lambda^{-1}\right) \varepsilon(L)\right) /\left(1-\lambda L^{-1}\right)=L^{-1} \lambda \varepsilon(\lambda)^{-1}\left[+\varepsilon_{r} L^{r}+\left(\varepsilon_{r-1}+\lambda \varepsilon_{r}\right) L^{r-1}\right.$

$$
\begin{align*}
& -L^{-1} \lambda I /\left(1-\lambda L^{-1}\right)+I /\left(1-\lambda L^{-1}\right) \\
& =I+\varepsilon(\lambda)^{-1}\left[\lambda \varepsilon_{r} L^{r-1}+\left(\lambda \varepsilon_{r-1}+\lambda^{2} \varepsilon_{r}\right) L^{r-2}+. .\left(\lambda \varepsilon_{1}+\lambda^{2} \varepsilon_{2}+\ldots \lambda^{r} \varepsilon_{r}\right)\right] \\
& =\varepsilon(\lambda)^{-1}\left[\varepsilon(\lambda)+\left(\lambda \varepsilon_{1}+\lambda^{2} \varepsilon_{2}+\ldots \lambda^{r} \varepsilon_{r}\right)\right. \\
& \left.+\lambda \varepsilon_{r} \mathrm{~L}^{\mathrm{r}-1}+\left(\lambda \varepsilon_{\mathrm{r}-1}+\lambda^{2} \varepsilon_{\mathrm{r}}\right) \mathrm{L}^{\mathrm{r}-2}+\ldots\left(\lambda \varepsilon_{2}+\lambda^{2} \varepsilon_{3}+\ldots \lambda^{\mathrm{r}-1} \varepsilon_{\mathrm{r}}\right) \mathrm{L}\right] \\
& =\varepsilon(\lambda)^{-1}\left[I+\left(\lambda \varepsilon_{2}+\lambda^{2} \varepsilon_{3}+\ldots \lambda^{r-1} \varepsilon_{r}\right) L\right. \\
& \left.+\ldots+\left(\lambda \varepsilon_{r-1}+\lambda^{2} \varepsilon_{r}\right) L^{r-2}+\lambda \varepsilon_{r} L^{r-1}\right] \\
& =\varepsilon(\lambda)^{-1}\left[I+\sum_{j=1}^{r-1}\left(\sum_{k=j+1}^{r} \lambda^{k-j} \varepsilon_{k}\right) L^{j}\right] \tag{17}
\end{align*}
$$

Using analogous, we get

$$
\begin{align*}
& \sum_{k=0}^{\infty}(\lambda)^{k} E_{t}\left(a_{t+k}\right)=\left[\left(I-L^{-1} \lambda \alpha(\lambda)^{-1} \alpha(L)\right) /\left(1-\lambda L^{-1}\right)\right] a_{t}  \tag{18}\\
& \text { and } \quad\left[I-L^{-1} \lambda \alpha(\lambda)^{-1} \alpha(L)\right] /\left(1-\lambda L^{-1}\right)=\alpha(\lambda)^{-1}\left[I+\sum_{j=1}^{q-1}\left(\sum_{k=j+1}^{q} \lambda^{\left.\left.k-j_{\alpha_{k}}\right) L^{j}\right]}\right.\right. \tag{19}
\end{align*}
$$

Thus, the explicit form of decision rule for $n_{t}$ is:

$$
\begin{align*}
n_{t}= & \rho_{1} n_{t-1}-\frac{\rho_{1}}{\delta} U \varepsilon(\lambda)^{-1}\left[I+\sum_{j=1}^{r-1}\left(\sum_{k=j+1}^{r} \lambda^{k-j} \varepsilon_{k}\right) L^{j}\right] x_{t} \\
& +\frac{\rho_{1}}{\delta} \alpha(\lambda)^{-1}\left[I+\sum_{j=1}^{q-1}\left(\sum_{k=j+1}^{q} \lambda^{k-j} \alpha_{k}\right) L^{j}\right] a_{t} \tag{20}
\end{align*}
$$

where

$$
\begin{aligned}
& \varepsilon(L) x_{t}=v_{t}^{x} \\
& \alpha(L) a_{t .}=v_{t}^{a}
\end{aligned}
$$

APPENDIX B. UNIT ROOT AND GRANGER-CAUSALITY TEST RESULTS

Table B.l. A unit root test of endogenous variables adjusting with linear trend

| Model: | $Y(t)=C+\alpha(1) Y(t-1)+\sum_{i=1}^{J} \gamma_{i} D_{i t}+e_{t} .$ <br> where $D_{i t}=Y(t-i)-Y(t-i-1) \quad$ for $i=1,2,3,4,5$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Y ( t ) | C | $\alpha(1)$ | $\gamma(1)$ | $\gamma(2)$ | $\gamma(3)$ | $\gamma(4)$ | $\gamma(5)$ | $\tau$ | MSE |
| A | $\begin{aligned} & -0.007 \\ & (2.3) \end{aligned}$ | $\begin{gathered} -0.34 \\ (0.12) \end{gathered}$ |  |  |  |  |  | $-11.5^{*}$ | 354.6 |
| A | $\begin{gathered} -0.42 \\ (0.3) \end{gathered}$ | $\begin{aligned} & -3.03 \\ & (0.05) \end{aligned}$ | $\begin{gathered} 2.03 \\ (0.04) \end{gathered}$ |  |  |  |  | -80.6* | 271.4 |
| A | $\begin{aligned} & -0.09 \\ & (0.21) \end{aligned}$ | $\begin{aligned} & 0.175 \\ & (0.3) \end{aligned}$ | $\begin{gathered} 0.38 \\ (0.23) \end{gathered}$ | $\begin{aligned} & -0.61 \\ & (0.15) \end{aligned}$ | $\begin{aligned} & -0.82 \\ & (0.08) \end{aligned}$ |  |  | - 2.75 | 2.97 |
| $\mathrm{SBHF}_{t}$ | $\begin{gathered} -6.17 \\ (20.9) \end{gathered}$ | $\begin{aligned} & -0.08 \\ & (0.12) \end{aligned}$ |  |  |  |  |  | - 9.* | 33,000. |
| $\mathrm{SBHF}_{t}$ | $\begin{gathered} -11.22 \\ (18.3) \end{gathered}$ | $\begin{aligned} & -0.63 \\ & (0.15) \end{aligned}$ | $\begin{aligned} & 0.5 \\ & (0.10) \end{aligned}$ |  |  |  |  | -10.9 * | 25,000. |
| $\mathrm{SBHF}_{t}$ | $\begin{gathered} -18.37 \\ (17.18) \end{gathered}$ | $\begin{aligned} & -1.28 \\ & (0.23) \end{aligned}$ | $\begin{gathered} 0.96 \\ (0.16) \end{gathered}$ | $\begin{gathered} 0.39 \\ (0.11) \end{gathered}$ |  |  |  | - 9.9* | 21,727. |
| $\mathrm{SBHF}_{t}$ | $\begin{array}{r} -1.07 \\ (9.2) \end{array}$ | $\begin{gathered} 0.69 \\ (0.19) \end{gathered}$ | $\begin{aligned} & -0.69 \\ & (0.15) \end{aligned}$ | $\begin{gathered} -0.77 \\ (0.11) \end{gathered}$ | $\begin{gathered} -0.88 \\ (0.06) \end{gathered}$ |  |  | - 1.6 | 6,145. |

*Significance at $\mathrm{c}=0.05$.

Table B. 1 - continued

| Model: | $\begin{aligned} & Y(t)=C+\alpha(1) Y(t-1)+\sum_{i=1}^{5} \gamma_{i} D_{i t}+e_{t} \\ & \text { where } D_{i t}=Y(t-i)-Y(t-i-1) \quad \text { for } i=1,2,3,4,5 \end{aligned}$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $Y(t)$ | C | $\alpha(1)$ | $\gamma(1)$ | $\gamma(2)$ | $\gamma(3)$ | $\gamma(4)$ | $\gamma(5)$ | $\tau$ | MSE |
| $\mathrm{SBHF}_{\mathbf{t}}$ | $\begin{gathered} -5.39 \\ (7.01) \end{gathered}$ | $\begin{gathered} 0.40 \\ (0.15) \end{gathered}$ | $\begin{gathered} 0.15 \\ (0.16) \end{gathered}$ | $\begin{aligned} & -0.003 \\ & (0.13) \end{aligned}$ | $\begin{gathered} -0.189 \\ (0.11) \end{gathered}$ | $\begin{gathered} 0.69 \\ (0.09) \end{gathered}$ |  | -4.0 * | 3,522. |
| $\mathrm{SBHF}_{t}$ | $\begin{aligned} & -3.07 \\ & (6.9) \end{aligned}$ | $\begin{gathered} 0.58 \\ (0.17) \end{gathered}$ | $\begin{gathered} 0.14 \\ (0.15) \end{gathered}$ | $\begin{gathered} -0.22 \\ (0.16) \end{gathered}$ | $\begin{aligned} & -0.36 \\ & (0.13) \end{aligned}$ | $\begin{gathered} 0.56 \\ (0.11) \end{gathered}$ | $\begin{aligned} & -0.26 \\ & (0.13) \end{aligned}$ | -2.47 | 3,364. |
| SBC | $\begin{gathered} -2.82 \\ (22.02) \end{gathered}$ | $\begin{aligned} & -0.14 \\ & (0.11) \end{aligned}$ |  |  |  |  |  | $-10.4 *$ | 36,370. |
| SBC | $\begin{gathered} -5.4 \\ (18.7) \end{gathered}$ | $\begin{aligned} & -0.75 \\ & (0.15) \end{aligned}$ | $\begin{gathered} 0.54 \\ (0.09) \end{gathered}$ |  |  |  |  | -11.6* | 26,234. |
| SBC | $\begin{gathered} -10.8 \\ (16.7) \end{gathered}$ | $\begin{aligned} & -1.59 \\ & (0.23) \end{aligned}$ | $\begin{gathered} 1.12 \\ (0.16) \end{gathered}$ | $\begin{gathered} 0.48 \\ (0.11) \end{gathered}$ |  |  |  | -11.3* | 20,767. |
| SBC | $\begin{array}{r} 0.68 \\ (8.4) \end{array}$ | $\begin{gathered} 0.67 \\ (0.19) \end{gathered}$ | $\begin{aligned} & -0.73 \\ & (0.15) \end{aligned}$ | $\begin{aligned} & -0.82 \\ & (0.10) \end{aligned}$ | $\begin{gathered} -0.89 \\ (0.06) \end{gathered}$ |  |  | -1.74 | 5,239. |
| SBC | $\begin{gathered} -1.79 \\ (7.4) \end{gathered}$ | $\begin{gathered} 0.48 \\ (0.17) \end{gathered}$ | $\begin{gathered} -0.12 \\ (0.19) \end{gathered}$ | $\begin{aligned} & -0.25 \\ & (0.15) \end{aligned}$ | $\begin{aligned} & -0.36 \\ & (0.13) \end{aligned}$ | $\begin{gathered} 0.49 \\ (0.11) \end{gathered}$ |  | -3.06 | 4,088. |
| SBC | $\begin{gathered} -2.41 \\ (7.5) \end{gathered}$ | $\begin{gathered} 0.42 \\ (0.19) \end{gathered}$ | $\begin{gathered} -0.11 \\ (0.19) \end{gathered}$ | $\begin{aligned} & -0.15 \\ & (0.19) \end{aligned}$ | $\begin{aligned} & -0.29 \\ & (0.16) \end{aligned}$ | $\begin{gathered} 0.59 \\ (0.14) \end{gathered}$ | $\begin{gathered} 0.09 \\ (0.13) \end{gathered}$ | -3.41* | 4,118. |

Table B. 1 - continued

```
Model: \(Y(t)=C+\alpha(1) Y(t-1)+\sum_{i=1}^{5} i_{i t}+e_{t}\)
    where \(D_{i t}=Y(t-i)-Y(t-i-1) \quad\) for \(i=1,2,3,4,5\)
```

| $\mathrm{Y}(\mathrm{t})$ | C | @(1) | $\gamma(1)$ | $\gamma(2)$ | $\gamma(3)$ | $\gamma(4)$ | $\gamma(5)$ | $\tau$ | MSE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| PM | $\begin{gathered} -0.49 \\ (2.85) \end{gathered}$ | $\begin{gathered} 0.75 \\ (0.08) \end{gathered}$ |  |  |  |  |  | - 3.125 | 552 |
| PM | $\begin{array}{r} 0.45 \\ (2.7) \end{array}$ | $\begin{gathered} 0.68 \\ (0.08) \end{gathered}$ | $\begin{gathered} 0.28 \\ (0.12) \end{gathered}$ |  |  |  |  | -4.0 * | 515. |
| PM | $\begin{aligned} & -0.44 \\ & (2.77) \end{aligned}$ | $\begin{gathered} 0.67 \\ (0.09) \end{gathered}$ | $\begin{gathered} 0.28 \\ (0.12) \end{gathered}$ | $\begin{gathered} 0.04 \\ (0.12) \end{gathered}$ |  |  |  | $-3.67^{*}$ | 522. |
| PM | $\begin{aligned} & -0.43 \\ & (2.79) \end{aligned}$ | $\begin{array}{r} 0.65 \\ (0.1) \end{array}$ | $\begin{gathered} 0.29 \\ (0.12) \end{gathered}$ | $\begin{gathered} 0.04 \\ (0.12) \end{gathered}$ | $\begin{gathered} 0.05 \\ (0.12) \end{gathered}$ |  |  | -3.46 * | 529. |
| PO | -0.08 | 0.82 |  |  |  |  |  | $-2.57$ | 10.4 |
| PO | $\begin{aligned} & -.08 \\ & (0.39) \end{aligned}$ | $\begin{gathered} 0.81 \\ (0.07) \end{gathered}$ | $\begin{aligned} & 0.4 \\ & (0.12) \end{aligned}$ |  |  |  |  | -2.7 | 10.5 |
| PO | $\begin{aligned} & -0.07 \\ & (0.39) \end{aligned}$ | $\begin{gathered} 0.79 \\ (0.07) \end{gathered}$ | $\begin{gathered} 0.05 \\ (0.12) \end{gathered}$ | $\begin{gathered} 0.17 \\ (0.12) \end{gathered}$ |  |  |  | - 3.0 | 10.4 |
| PO | $\begin{aligned} & -0.07 \\ & (0.39) \end{aligned}$ | $\begin{gathered} 0.77 \\ (0.07) \end{gathered}$ | $\begin{gathered} 0.06 \\ (0.12) \end{gathered}$ | $\begin{gathered} 0.17 \\ (0.12) \end{gathered}$ | $\begin{gathered} 0.07 \\ (0.12) \end{gathered}$ |  |  | -3.28 * | 10.5 |
| PS | $\begin{gathered} -0.02 \\ (0.09) \end{gathered}$ | $\begin{gathered} 0.72 \\ (0.08) \end{gathered}$ |  |  |  |  |  | -3.3 | 0.63 |

Table B. 1 - continued

| Model: | $Y(t)=C+(1) Y(t-1)+\sum_{i=1}^{5} \gamma_{i} D_{i t}+e_{t}$ <br> where: $D_{i t}=Y(t-i)-Y(t-i-1) \quad$ for $i=1,2,3,4,5$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Y ( t ) | C | $\alpha(1)$ | $\gamma(1)$ | $r(2)$ | $\gamma$ ¢ 3 ) | $\gamma(4)$ | $\gamma(5)$ | $\tau$ | MSE |
| PS | $\begin{aligned} & -0.025 \\ & (0.09) \end{aligned}$ | $\begin{gathered} 0.69 \\ (0.09) \end{gathered}$ | $\begin{gathered} 0.126 \\ (0.12) \end{gathered}$ |  |  |  |  | -3.49* | 0.63 |
| SC | $\begin{gathered} -0.07 \\ (1.81) \end{gathered}$ | $\begin{gathered} 0.46 \\ (0.11) \end{gathered}$ |  |  |  |  |  | $-4.9 *$ | 223.8 |
| SC | -0.27 | 0.23 | 0.44 |  |  |  |  | -6.59* | 185. |
| SC | $\begin{gathered} -0.21 \\ (1.65) \end{gathered}$ | $\begin{gathered} 0.32 \\ (0.15) \end{gathered}$ | $\begin{aligned} & 0.4 \\ & (0.12) \end{aligned}$ | $\begin{gathered} -0.12 \\ (0.13) \end{gathered}$ |  |  |  | -4.5* | 185. |

Table B.2. A unit root test of exogenous variables (all variables are adjusted with linear trend)

*Significance at $\alpha=0.05$.

Table B. 2 - continued


Table B. 2 - continued

| Model: | $\begin{aligned} & X(t)=C+\beta(1) X(t-1)+\gamma_{1} D_{1 t}+\gamma_{2} D_{2 t}+\gamma_{3} D_{3 t}+\gamma_{4} D_{4 t}+e_{t} \\ & \text { where } D_{i t}=X(t-i)-X(t-i-1) \quad \text { for } i=1,2,3,4,5 \end{aligned}$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{X}(\mathrm{t})$ | C | $\alpha(1)$ | $\gamma(1)$ | $\gamma(2)$ | $\gamma(3)$ | $\gamma(4)$ | $\gamma(5)$ | $\tau$ | MSE |
| SOH | $\begin{aligned} & -29.8 \\ & (37.1) \end{aligned}$ | $\begin{gathered} 0.278 \\ (0.17) \end{gathered}$ | $\begin{gathered} 0.54 \\ (0.27) \end{gathered}$ | $\begin{gathered} 0.04 \\ (0.28) \end{gathered}$ |  |  |  | -4.27 | 114,054. |
| SOH | $\begin{gathered} -23.5 \\ (36.4) \end{gathered}$ | $\begin{gathered} 0.44 \\ (0.18) \end{gathered}$ | $\begin{gathered} 0.38 \\ (0.3) \end{gathered}$ | $\begin{gathered} 0.008 \\ (0.28) \end{gathered}$ | $\begin{aligned} & -0.58 \\ & (0.28) \end{aligned}$ |  |  | -3.11 | 109,451. |
| SMH | $\begin{gathered} -15.65 \\ (15.8) \end{gathered}$ | $\begin{gathered} 1.03 \\ (0.19) \end{gathered}$ |  |  |  |  |  | -0.16 | 21,018. |
| SMH | $\begin{gathered} -15.37 \\ (15.8) \end{gathered}$ | $\begin{gathered} 0.99 \\ (0.09) \end{gathered}$ | $\begin{array}{r} 0.216 \\ (0.34) \end{array}$ |  |  |  | - | -0.05 | 21,172. |
| SOX | $\begin{gathered} -17.3 \\ (18.2) \end{gathered}$ | $\begin{array}{r} 0.83 \\ (0.2) \end{array}$ | $\begin{gathered} 0.4 \\ (0.2) \end{gathered}$ | $\begin{aligned} & -0.4 \\ & (0.2) \end{aligned}$ | $\begin{array}{r} 0.34 \\ (0.2) \end{array}$ |  |  | -0.85 | 28,030. |
| COOP | $\begin{aligned} & -0.09 \\ & (0.41) \end{aligned}$ | $\begin{gathered} 0.74 \\ (0.08) \end{gathered}$ |  |  |  |  |  | -3.25 | 11.4 |
| COOP | $\begin{gathered} -0.09 \\ (0.4) \end{gathered}$ | $\begin{gathered} 0.79 \\ (0.08) \end{gathered}$ | $\begin{aligned} & -0.21 \\ & (0.12) \end{aligned}$ |  |  |  |  | -2.6 | 11.03 |
| COOP | $\begin{aligned} & -0.04 \\ & (0.37) \end{aligned}$ | $\begin{gathered} 0.71 \\ (0.08) \end{gathered}$ | $\begin{aligned} & -0.12 \\ & (0.12) \end{aligned}$ | $\begin{gathered} 0.19 \\ (0.12) \end{gathered}$ | $\begin{gathered} 0.45 \\ (0.11) \end{gathered}$ |  |  | -3.6 * | 9.11 |
| CRNO | $\begin{aligned} & -0.16 \\ & (0.46) \end{aligned}$ | $\begin{gathered} 0.76 \\ (0.08) \end{gathered}$ |  |  |  |  |  | -3.0 | 14.6 |

Table B. 2 - continued

| Model:$X(t)$ | $\begin{aligned} & X(t)=C+\beta(1) X(t-1)+\gamma_{1} D_{1 t}+\gamma_{2} D_{2 t}+\gamma_{3} D_{3 t}+\gamma_{4} D_{2 t}+\gamma_{5} D_{4 t}+e_{t} \\ & \text { where } D_{i t}=X(t-i)-X(t-i-1) \quad \text { for } i=1,2,3,4,5 \end{aligned}$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | C | $\alpha(1)$ | $\gamma(1)$ | $\gamma(2)$ | $\gamma(3)$ | $\gamma(4)$ | $\gamma(5)$ | $\tau$ | MSE |
| CRNO | -0.16 | 0.76 | 0.03 |  |  |  |  | -3.0 | 14.8 |
| TB | $\begin{gathered} 0.53 \\ (0.27) \end{gathered}$ | $\begin{gathered} 0.91 \\ (0.05) \end{gathered}$ |  |  |  |  |  | -1.8 | 0.34 |
| TB | $\begin{gathered} 0.62 \\ (0.26) \end{gathered}$ | $\begin{gathered} 0.89 \\ (0.05) \end{gathered}$ | $\begin{gathered} 0.32 \\ (0.12) \end{gathered}$ |  |  |  |  | -2.2 | 0.31 |
| TB | $\begin{gathered} 0.55 \\ (0.26) \end{gathered}$ | $\begin{gathered} 6.90 \\ (0.05) \end{gathered}$ | $\begin{gathered} 0.35 \\ (0.12) \end{gathered}$ | $\begin{aligned} & -0.13 \\ & (0.13) \end{aligned}$ |  |  |  | $-2.0$ | 0.31 |
| TB | $\begin{gathered} 0.64 \\ (0.26) \end{gathered}$ | $\begin{gathered} 0.88 \\ (0.13) \end{gathered}$ | $\begin{gathered} 0.41 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.22 \\ & (0.13) \end{aligned}$ | $\begin{gathered} 0.3 \\ (0.12) \end{gathered}$ |  |  | -2.4 | 0.29 |

Table B3. Cumulative "t-like" distribution ${ }^{\text {a }}$

|  | Probability of a smaller value |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Sample size | 0.01 | 0.025 | 0.05 | 0.10 |  |
| 50 | -4.15 | -3.80 | -3.5 | -3.18 |  |
| 100 | -4.04 | -3.73 | -3.45 | -3.15 |  |
| 250 | -3.99 | -3.69 | -3.43 | -3.13 |  |
| 500 | -3.98 | -3.68 | -3.42 | -3.13 |  |
| $\infty$ | -3.96 | -3.66 | -3.41 | -3.12 |  |

Table B.4. Granger-causality test results

| Mode1: <br> Y | $\begin{aligned} Y(t)= & C+\sum_{s=1}^{4} \alpha(s) Y(t-s)+ \\ & \gamma_{3} \operatorname{Spri}+e_{t} \end{aligned}$ |  | $\sum_{k=1}^{6} \beta(k) X(t-k)+\gamma_{1} \text { Eall }+\gamma_{2} \text { Wint }+$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | X | $\begin{gathered} c \\ (\mathrm{~s} \cdot \mathrm{e}) \end{gathered}$ | $\alpha(1)$ | $\alpha(2)$ | $\alpha$ (3) | $\alpha(4)$ |
| A | CORPF | $\begin{aligned} & -0.49 \\ & (0.26)^{a} \end{aligned}$ | $\begin{aligned} & -0.15 \\ & (0.08) \end{aligned}$ | $\begin{aligned} & -0.18 \\ & (0.08) \end{aligned}$ | $\begin{aligned} & -0.15 \\ & (0.08) \end{aligned}$ | $\begin{gathered} 0.83 \\ (0.08) \end{gathered}$ |
| CORPF | A | $\begin{aligned} & -0.003 \\ & (0.03) \end{aligned}$ | $\begin{gathered} 1.0 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.17 \\ & (0.19) \end{aligned}$ | $\begin{gathered} 0.28 \\ (0.19) \end{gathered}$ | $\begin{aligned} & -0.24 \\ & (0.14) \end{aligned}$ |
| A | SBX | $\begin{aligned} & -0.06 \\ & (0.33) \end{aligned}$ | $\begin{aligned} & -0.17 \\ & (0.09) \end{aligned}$ | $\begin{aligned} & -0.11 \\ & (0.1) \end{aligned}$ | $\begin{aligned} & -0.17 \\ & (0.09) \end{aligned}$ | $\begin{gathered} 0.77 \\ (0.1) \end{gathered}$ |
| SBX | A | -3.92 | 0.28 | 0.25 | -0.22 | 0.05 |
| A | HPAU | $\begin{aligned} & -0.34 \\ & (0.27) \end{aligned}$ | $\begin{aligned} & -0.11 \\ & (0.08) \end{aligned}$ | $\begin{aligned} & -0.13 \\ & (0.08) \end{aligned}$ | $\begin{aligned} & -0.15 \\ & (0.08) \end{aligned}$ | $\begin{gathered} 0.84 \\ (0.08) \end{gathered}$ |
| HPAU | A | $\begin{aligned} & -0.05 \\ & (0.08) \end{aligned}$ | $\begin{gathered} 0.93 \\ (0.12) \end{gathered}$ | $\begin{aligned} & -0.17 \\ & (0.15) \end{aligned}$ | $\begin{gathered} 0.24 \\ (0.15) \end{gathered}$ | $\begin{aligned} & -0.20 \\ & (0.12) \end{aligned}$ |
| A | COOP | $\begin{aligned} & -0.39 \\ & (0.26) \end{aligned}$ | $\begin{aligned} & -0.12 \\ & (0.08) \end{aligned}$ | $\begin{aligned} & -0.14 \\ & (0.08) \end{aligned}$ | $\begin{aligned} & -0.13 \\ & (0.08) \end{aligned}$ | $\begin{gathered} 0.84 \\ (0.08) \end{gathered}$ |
| COOP | A | $\begin{aligned} & -0.0001 \\ & (0.45) \end{aligned}$ | $\begin{gathered} 0.59 \\ (0.12) \end{gathered}$ | $\begin{gathered} 0.37 \\ (0.14) \end{gathered}$ | $\begin{gathered} 0.19 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.46 \\ & (0.12) \end{aligned}$ |
| A | SOH | $\begin{aligned} & -0.3 \\ & (0.29) \end{aligned}$ | $\begin{aligned} & -0.16 \\ & (0.08) \end{aligned}$ | $\begin{aligned} & -0.15 \\ & (0.08) \end{aligned}$ | $\begin{aligned} & -0.15 \\ & (0.08) \end{aligned}$ | $\begin{gathered} 0.82 \\ (0.08) \end{gathered}$ |
| SOH | A | $\begin{aligned} & -8.48 \\ & (15.3) \end{aligned}$ | $\begin{gathered} 1.09 \\ (0.15) \end{gathered}$ | $\begin{aligned} & -0.02 \\ & (0.21) \end{aligned}$ | $\begin{aligned} & -0.31 \\ & (0.2) \end{aligned}$ | $\begin{gathered} 0.05 \\ (0.13) \end{gathered}$ |

[^6]| $\beta(1)$ | $\beta(2)$ | $\beta(3)$ | $\beta(4)$ | $\beta(5)$ | $\beta(6)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| -0.77 | 0.06 | -1.92 | 2.04 | 2.47 | -3.37 |
| $(1.2)$ | $(1.7)$ | $(1.7)$ | $(1.7)$ | $(1.9)$ | $(1.5)$ |
| -0.02 | 0.01 | 0.008 | 0.002 | 0.03 | -0.01 |
| $(0.01)$ | $(0.01)$ | $(0.009)$ | $(0.009)$ | $(0.01)$ | $(0.01)$ |
| 0.01 | 0.001 | 0.002 | 0.01 | 0.02 | 0.001 |
| $(0.01)$ | $(0.02)$ | $(0.02)$ | $(0.02)$ | $(0.02)$ | $(0.02)$ |
|  |  |  |  |  |  |
| -3.14 | 2.04 | 1.16 | 0.25 | 0.97 | -1.89 |
| -0.31 | -0.30 | 0.52 | 0.33 | -0.09 | -0.13 |
| $(0.44)$ | $(0.54)$ | $(0.53)$ | $(0.52)$ | $(0.52)$ | $(0.38)$ |
| 0.005 | 0.06 | -0.01 | 0.03 | -0.07 | -0.07 |
| $(0.04)$ | $(0.04)$ | $(0.02)$ | $(0.02)$ | $(0.04)$ | $(0.04)$ |
| -0.05 | -0.02 | -0.08 | 0.11 | 0.06 | -0.13 |
| $(0.08)$ | $(0.09)$ | $(0.08)$ | $(0.08)$ | $(0.09)$ | $(0.08)$ |
| 0.15 | -0.43 | 0.07 | 0.08 | -0.12 | 0.47 |
| $(0.24)$ | $(0.24)$ | $(0.14)$ | $(0.14)$ | $(0.24)$ | $(0.24)$ |
| -0.0003 | -0.0002 | 0.001 | 0.002 | -0.002 | 0.001 |
| $(0.002)$ | $(0.004)$ | $(0.004)$ | $(0.004)$ | $(0.004)$ | $(0.003)$ |
| -17.95 | 14.38 | -1.97 | 2.89 | 20.51 | -12.17 |
| $(8.11)$ | $(0.9)$ | $(4.5)$ | $(4.5)$ | $(8.2)$ | $(8.8)$ |
|  |  |  |  |  |  |

Table B. 4 - continued

| Мо | $\begin{aligned} Y(t)= & C+\sum_{s=1}^{4} \alpha(s) Y(t-s)+ \\ & \gamma_{3} \text { Spri }+e_{t} \end{aligned}$ |  | $\sum_{k=1}^{6} \beta(k) X(t-k)+\gamma_{1} \text { Fall }+\gamma_{2} \text { Wint }+$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Fall | Wint | Spri | $\mathrm{F}_{6,50}$ | $\mathrm{R}^{2}$ | MSE |
| $\begin{array}{r} 0.47 \\ (1.9) \end{array}$ | $\begin{aligned} & -0.75 \\ & (1.8) \end{aligned}$ | $\begin{gathered} 1.74 \\ (1.8) \end{gathered}$ | 1.95* | 0.99 | 3.04 |
| $\begin{gathered} -0.04 \\ (0.2) \end{gathered}$ | $\begin{aligned} & -0.06 \\ & (0.2) \end{aligned}$ | $\begin{array}{r} 0.15 \\ (0.2) \end{array}$ | 1.15 | 0.84 | 0.04 |
| $\begin{aligned} & -2.35 \\ & (2.74) \end{aligned}$ | $\begin{aligned} & -1.1 \\ & (2.55) \end{aligned}$ | $\begin{gathered} 3.69 \\ (2.67) \end{gathered}$ | 0.53 | 0.99 | 3.52 |
| 3.63 | -45.4 | -22.02 | 3.67* | 0.67 | 273.4 |
| $\begin{aligned} & -3.79 \\ & (3.14) \end{aligned}$ | $\begin{gathered} 1.91 \\ (3.4) \end{gathered}$ | $\begin{gathered} 5.62 \\ (3.0) \end{gathered}$ | 0.67 | 0.99 | 3.47 |
| $\begin{gathered} 3.61 \\ (0.86) \end{gathered}$ | $\begin{aligned} & -1.24 \\ & (0.92) \end{aligned}$ | $\begin{aligned} & -5.44 \\ & (0.97) \end{aligned}$ | 4.62* | 0.96 | 0.28 |
| $\begin{aligned} & -0.41 \\ & (1.88) \end{aligned}$ | $\begin{aligned} & -0.75 \\ & (1.97) \end{aligned}$ | $\begin{gathered} 2.32 \\ (1.87) \end{gathered}$ | 1.3 | 0.99 | 3.2 |
| $\begin{gathered} 0.32 \\ (3.3) \end{gathered}$ | $\begin{aligned} & -0.61 \\ & (3.3) \end{aligned}$ | $\begin{aligned} & -0.34 \\ & (3.3) \end{aligned}$ | 0.77 | 0.70 | 9.8 |
| $\begin{aligned} & -1.1 \\ & (2.08) \end{aligned}$ | $\begin{aligned} & -1.03 \\ & (1.96) \end{aligned}$ | $\begin{gathered} 2.58 \\ (2.04) \end{gathered}$ | 0.72 | 0.99 | 3.45 |
| $\begin{array}{r} 54.96 \\ (106.4) \end{array}$ | $\begin{array}{r} 174.72 \\ (102.9) \end{array}$ | $\begin{aligned} & -114.08 \\ & (105.8) \end{aligned}$ | 2.09** | 0.84 | 9,678.2 |

Table B. 4 - continued

| $\mathrm{Mo}$ | $\begin{gathered} Y(t)=C+\sum_{\sum_{s=1}^{4} \alpha(s) Y(t-s)}^{\gamma_{3} \operatorname{spri}+e_{t}} \end{gathered}$ |  | $\sum_{k=1}^{6} \beta(k) X(t-k)+\gamma_{1} \text { Fall }+\gamma_{2} \text { Wint }+$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Y | X | $\begin{gathered} c \\ (\mathrm{~s} \cdot \mathrm{e}) \end{gathered}$ | $\alpha(1)$ | $\alpha$ (2) | $\alpha$ (3) | $\alpha$ (4) |
| A | SMH | $\begin{aligned} & -.30 \\ & (0.26) \end{aligned}$ | $\begin{aligned} & -0.19 \\ & (0.08) \end{aligned}$ | $\begin{aligned} & -0.22 \\ & (0.08) \end{aligned}$ | $\begin{aligned} & -0.21 \\ & (0.08) \end{aligned}$ | $\begin{gathered} 0.48 \\ (0.08) \end{gathered}$ |
| SMH | A | $\begin{gathered} 5.13 \\ (6.5) \end{gathered}$ | $\begin{gathered} 1.09 \\ (0.15) \end{gathered}$ | $\begin{aligned} & -0.26 \\ & (0.22) \end{aligned}$ | $\begin{gathered} 0.13 \\ (0.22) \end{gathered}$ | $\begin{aligned} & -0.12 \\ & (0.14) \end{aligned}$ |
| A | TB <br> (not detrend) | $\begin{gathered} 1.46 \\ (0.93) \end{gathered}$ | $\begin{aligned} & -0.24 \\ & (0.09) \end{aligned}$ | $\begin{aligned} & -0.24 \\ & (0.09) \end{aligned}$ | $\begin{aligned} & -0.23 \\ & (0.09) \end{aligned}$ | $\begin{gathered} 0.72 \\ (0.09) \end{gathered}$ |
| TB | A | $\begin{gathered} 0.59 \\ (0.25) \end{gathered}$ | $\begin{gathered} 1.26 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.44 \\ & (0.22) \end{aligned}$ | $\begin{gathered} 0.29 \\ (0.22) \end{gathered}$ | $\begin{aligned} & -0.22 \\ & (0.14) \end{aligned}$ |
| A | FIMP | $\begin{aligned} & -0.29 \\ & (0.26) \end{aligned}$ | $\begin{aligned} & -0.14 \\ & (0.07) \end{aligned}$ | $\begin{aligned} & -0.10 \\ & (0.08) \end{aligned}$ | $\begin{aligned} & -0.12 \\ & (0.08) \end{aligned}$ | $\begin{gathered} 0.85 \\ (0.08) \end{gathered}$ |
| FIMP | A | $\begin{aligned} & -4.6 .7 \\ & (5.04) \end{aligned}$ | $\begin{gathered} 1.13 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.17 \\ & (0.2) \end{aligned}$ | $\begin{gathered} 0.03 \\ (0.18) \end{gathered}$ | $\begin{aligned} & -0.28 \\ & (0.13) \end{aligned}$ |
| SBHF | CORPF | $\begin{gathered} -13.8 \\ (8.4) \end{gathered}$ | $\begin{gathered} 0.38 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.06 \\ & (0.13) \end{aligned}$ | $\begin{aligned} & -0.32 \\ & (0.13) \end{aligned}$ | $\begin{gathered} 0.41 \\ (0.14) \end{gathered}$ |
| CORPF | SBHF | $\begin{aligned} & -0.04 \\ & (0.03) \end{aligned}$ | $\begin{gathered} 0.84 \\ (0.15) \end{gathered}$ | $\begin{gathered} 0.004 \\ (0.19) \end{gathered}$ | $\begin{gathered} 0.23 \\ (0.2) \end{gathered}$ | $\begin{aligned} & -0.2 \\ & (0.14) \end{aligned}$ |
| SBHF | SBX | $\begin{gathered} -20.8 \\ (9.4) \end{gathered}$ | $\begin{gathered} 0.5 \\ (0.13) \end{gathered}$ | $\begin{gathered} 0.13 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.49 \\ & (0.14) \end{aligned}$ | $\begin{gathered} 0.28 \\ (0.14) \end{gathered}$ |
| SBX | SBHF | $\begin{aligned} & -5.9 \\ & (3.37) \end{aligned}$ | $\begin{gathered} 0.15 \\ (0.14) \end{gathered}$ | $\begin{gathered} 0.14 \\ (0.16) \end{gathered}$ | $\begin{aligned} & -0.09 \\ & (0.16) \end{aligned}$ | $\begin{gathered} 0.04 \\ (0.16) \end{gathered}$ |
| SBHF | HPAU | $\begin{array}{r} -16.90 \\ (8.5) \end{array}$ | $\begin{gathered} 0.35 \\ (0.12) \end{gathered}$ | $\begin{aligned} & -0.09 \\ & (0.13) \end{aligned}$ | $\begin{aligned} & -0.31 \\ & (0.13) \end{aligned}$ | $\begin{gathered} 0.42 \\ (0.14) \end{gathered}$ |
| HPAU | SBHF | $\begin{aligned} & -0.07 \\ & (0.09) \end{aligned}$ | $\begin{gathered} 0.92 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.17 \\ & (0.16) \end{aligned}$ | $\begin{gathered} 0.19 \\ (0.16) \end{gathered}$ | $\begin{aligned} & -0.17 \\ & (0.12) \end{aligned}$ |
| SBHF | COOP | $\begin{gathered} -12.9 \\ (8.5) \end{gathered}$ | $\begin{gathered} 0.36 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.07 \\ & (0.13) \end{aligned}$ | $\begin{aligned} & -0.34 \\ & (0.13) \end{aligned}$ | $\begin{gathered} 0.46 \\ (0.13) \end{gathered}$ |


| $\beta(1)$ | $\beta(2)$ | $\beta(3)$ | $\beta(4)$ | $\beta(5)$ | $\beta(6)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| -0.002 | -0.005 | 0.002 | 0.01 | -0.007 | -0.007 |
| $(0.006)$ | $(0.009)$ | $(0.009)$ | $(0.008)$ | $(0.008)$ | $(0.006)$ |
| -2.53 | 5.13 | 0.74 | 1.29 | 4.43 | -2.6 |
| $(3.45)$ | $(3.6)$ | $(2.1)$ | $(2.1)$ | $(3.58)$ | $(3.6)$ |
| 0.12 | 0.44 | -1.08 | 1.17 | -1.43 | 0.36 |
| $(0.47)$ | $(0.76)$ | $(0.79)$ | $(0.79)$ | $(0.79)$ | $(0.52)$ |
| 0.04 | 0.02 | -0.03 | 0.001 | -0.04 | -0.04 |
| $(0.04)$ | $(0.02)$ | $(0.02)$ | $(0.02)$ | $(0.04)$ | $(0.04)$ |
| 0.02 | -0.01 | -0.003 | 0.0008 | 0.0003 | 0.004 |
| $(0.006)$ | $(0.009)$ | $(0.009)$ | $(0.009)$ | $(0.01)$ | $(0.008)$ |
|  |  |  |  |  |  |
| -9.28 | -0.78 | -1.09 | -1.75 | 7.35 | -0.29 |
| $(2.9)$ | $(3.25)$ | $(1.56)$ | $(0.55)$ | $(3.0)$ | $(3.2)$ |
| 4.58 | 5.13 | -19.8 | -0.57 | 67.53 | -46.5 |
| $(40.87)$ | $(57.9)$ | $(56.3)$ | $(55.22)$ | $(63.3)$ | $(47.9)$ |
| -0.0005 | -0.0005 | 0.0001 | -0.0001 | -0.0008 | -0.0006 |
| $(0.0005)$ | $(0.0005)$ | $(0.0005)$ | $(0.6005)$ | $(0.0007)$ | $(0.0007)$ |
| -1.47 | 0.41 | 0.81 | -1.17 | -0.22 | 0.58 |
| $(0.37)$ | $(0.42)$ | $(0.41)$ | $(0.42)$ | $(0.42)$ | $(0.38)$ |
| 0.02 | 0.11 | -0.18 | 0.03 | 0.05 | -0.10 |
| $(0.05)$ | $(0.06)$ | $(0.05)$ | $(0.05)$ | $(0.06)$ | $(0.05)$ |
| -3.26 | 11.25 | -22.89 | -5.13 | -4.37 | 18.05 |
| $(13.4)$ | $(15.6)$ | $(14.9)$ | $(15.4)$ | $(15.1)$ | $(11.1)$ |
| -0.001 | 0.004 | -0.006 | 0.001 | 0.002 | -0.001 |
| $(0.001$ | $(0.001)$ | $(0.001)$ | $(0.001)$ | $(0.001)$ | $(0.001)$ |
| 1.22 | 0.6 | -0.74 | 2.36 | -1.03 | -0.29 |
| $(2.57)$ | $(3.06)$ | $(2.85)$ | $(2.83)$ | $(3.04)$ | $(2.7)$ |
|  |  |  |  |  |  |

Table B. 4 - continued

| Model: <br> Fall | $\begin{aligned} Y(t)= & C+\sum_{s=1}^{4} \alpha(s) Y(t-s)+ \\ & \gamma_{3} S p r i+e_{t} \end{aligned}$ |  | $\sum_{k=1}^{6} \beta(k) X(t-k)+\gamma_{1} \text { Fall }+\gamma_{2} \text { Wint }+$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Wint | Spri | $\mathrm{F}_{6,50}$ | $\mathrm{R}^{2}$ | MSE |
| $\begin{gathered} 0.04 \\ (1.89) \end{gathered}$ | $\begin{aligned} & -0.98 \\ & (1.87) \end{aligned}$ | $\begin{gathered} 1.76 \\ (1.89) \end{gathered}$ | 1.6 | 0.99 | 3.14 |
| $\begin{aligned} & -25.5 \\ & (47.0) \end{aligned}$ | $\begin{gathered} 47.3 \\ (47.4) \end{gathered}$ | $\begin{array}{r} 21.33 \\ (47.5) \end{array}$ | 0.73 | 0.80 | 2,012. |
| $\begin{aligned} & -0.63 \\ & (1.92) \end{aligned}$ | $\begin{aligned} & 01.45 \\ & (1.88) \end{aligned}$ | $\begin{gathered} 2.8 \\ (1.96) \end{gathered}$ | $2.17 * *$ | 0.99 | 2.97 |
| $\begin{gathered} 0.17 \\ (0.56) \end{gathered}$ | $\begin{gathered} 0.8 \\ (0.54) \end{gathered}$ | $\begin{aligned} & -0.57 \\ & (0.55) \end{aligned}$ | 1.19 | 0.9 | 0.26 |
| $\begin{gathered} -1.5 \\ 1.88 \end{gathered}$ | $\begin{array}{r} -0.84 \\ 1.89 \end{array}$ | $\begin{aligned} & 2.47 \\ & 1.87 \end{aligned}$ | 1.58 | 0.99 | 3.15 |
| $\begin{array}{r} -14.15 \\ (36.6) \end{array}$ | $\begin{gathered} -14.39 \\ (36.3) \end{gathered}$ | $\begin{gathered} 2.7 \\ (36.4) \end{gathered}$ | 1.81 | 0.81 | 1,194. |
| $\begin{aligned} & 194.3 \\ & (32.7) \end{aligned}$ | $\begin{aligned} & -89.14 \\ & (41.2) \end{aligned}$ | $\begin{array}{r} -110.7 \\ (39.9) \end{array}$ | 0.36 | 0.89 | 3,320. |
| $\begin{aligned} & -0.21 \\ & (0.16) \end{aligned}$ | $\begin{aligned} & -0.05 \\ & (0.16) \end{aligned}$ | $\begin{gathered} 0.25 \\ (0.15) \end{gathered}$ | 1.65 | 0.85 | 0.04 |
| $\begin{aligned} & 226.4 \\ & (27.6) \end{aligned}$ | $\begin{aligned} & -48.6 \\ & (43.7) \end{aligned}$ | $\begin{array}{r} -165.2 \\ (3.53) \end{array}$ | $5.17{ }^{*}$ | 0.93 | 2,135. |
| $\begin{gathered} 29.5 \\ (15.7) \end{gathered}$ | $\begin{aligned} & -21.8 \\ & (16.7) \end{aligned}$ | $\begin{aligned} & -28.7 \\ & (14.9) \end{aligned}$ | 2.57 * | 0.63 | 301. |
| $\begin{aligned} & 291.3 \\ & (64.9) \end{aligned}$ | $\begin{array}{r} -100.7 \\ (87.9) \end{array}$ | $\begin{gathered} -211.7 \\ (71.5) \end{gathered}$ | 1.35 | 0.91 | 2,978. |
| $\begin{gathered} 3.8 \\ (0.78) \end{gathered}$ | $\begin{aligned} & -1.6 \\ & (0.89) \end{aligned}$ | $\begin{aligned} & -4.6 \\ & (0.73) \end{aligned}$ | 3.2 * | 0.95 | 0.32 |
| $\begin{aligned} & 186.3 \\ & (31.7) \end{aligned}$ | $\begin{aligned} & -94.7 \\ & (40.9) \end{aligned}$ | $\begin{aligned} & -107.6 \\ & (39.5) \end{aligned}$ | 0.52 | 0.89 | 3,261. |

Table B. 4 - continued


| $\beta(1)$ | $\beta(2)$ | $\beta(3)$ | $\beta(4)$ | $\beta(5)$ | $\beta(6)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0003 | 0.01 | -0.005 | 0.002 | -0.009 | -0.01 |
| $(0.009)$ | $(0.009)$ | $(0.007)$ | $(0.008)$ | $(0.00)$ | $(0.01)$ |
| -0.06 | -0.09 | 0.09 | -0.07 | 0.09 | -0.02 |
| $(0.08)$ | $(0.11)$ | $(0.11)$ | $(0.11)$ | $(0.11)$ | $(0.07)$ |
| 0.07 | 0.46 | 0.03 | 0.05 | -0.26 | -0.01 |
| $(0.32)$ | $(0.33)$ | $(0.27)$ | $(0.29)$ | $(0.38)$ | $(0.34)$ |
| -0.05 | -0.21 | 0.42 | -0.27 | 0.29 | -0.15 |
| $(0.18)$ | $(0.28)$ | $(0.28)$ | $(0.27)$ | $(0.27)$ | $(0.19)$ |
| -0.11 | 0.29 | -0.05 | 0.07 | -0.01 | -0.41 |
| $(0.11)$ | $(0.12)$ | $(0.09)$ | $(0.01)$ | $(0.13)$ | $(0.12)$ |
| 27.02 | -26.03 | -13.2 | 24.16 | 9.24 | -18.1 |
| $(14.7)$ | $(24.5)$ | $(24.9)$ | $(24.8)$ | $(24.7)$ | $(14.7)$ |
| -0.001 | 0.00004 | 0.0005 | -0.001 | -0.002 | 0.001 |
| $(0.001)$ | $(0.001)$ | $(0.001)$ | $(0.001)$ | $(0.002)$ | $(0.001)$ |
| -0.53 | 0.73 | -0.09 | -0.06 | -0.02 | 0.02 |
| $(0.2)$ | $(0.3)$ | $(0.3)$ | $(0.3)$ | $(0.3)$ | $(0.2)$ |
| -0.05 | 0.03 | -0.08 | 0.02 | 0.02 | -0.15 |
| $(0.12)$ | $(0.12)$ | $(0.09)$ | $(0.1)$ | $(0.14)$ | $(0.12)$ |
| -26.6 | -21.6 | 2.6 | 50.8 | 8.4 | -23.7 |
| $(43.9)$ | $(61.3)$ | $(60.6)$ | $(60.1)$ | $(66.6)$ | $(50.3)$ |
| 0.002 | 0.0001 | -0.0003 | -0.0001 | -0.001 | -0.0003 |
| $(0.0005)$ | $(0.0005)$ | $(0.0004)$ | $(0.0004)$ | $(0.0005)$ | $(0.0005)$ |
| -1.1 | -0.29 | -1.17 | -0.71 | -0.49 | 0.5 |
| 0.01 | 0.07 | -0.002 | -0.0007 | 0.04 | -0.06 |
| $(0.05)$ | $(0.05)$ | $(0.04)$ | $(0.04)$ | $(0.04)$ | $(0.04)$ |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

Table B. 4 - continued

| Mo | $\begin{aligned} Y(t)= & C+\sum_{\mathbf{s}=1} \alpha(s) \\ & \gamma_{3} \operatorname{Spri}=e_{t} \end{aligned}$ |  | $\sum_{k=1}^{6} \beta(k) X(t-k)+\gamma_{1} \text { Fall }+\gamma_{2} \text { Wint }+$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Fall | Wint | Spri | ${ }^{5} 6,50$ | $\mathrm{R}^{2}$ | MSE |
| $\begin{aligned} & -3.15 \\ & (2.35) \end{aligned}$ | $\begin{gathered} 0.76 \\ (2.41) \end{gathered}$ | $\begin{gathered} 1.59 \\ (2.26) \end{gathered}$ | 1.75 | 0.73 | 8.86 |
| $\begin{aligned} & 160.7 \\ & (35.15) \end{aligned}$ | $\begin{aligned} & -96.4 \\ & (39.0) \end{aligned}$ | $\begin{aligned} & -90.4 \\ & (39.2) \end{aligned}$ | 1.57 | 0.90 | 2,914.9 |
| $\begin{aligned} & 113.5 \\ & (89.2) \end{aligned}$ | $\begin{aligned} & 129.6 \\ & (86.9) \end{aligned}$ | $\begin{aligned} & -98.8 \\ & (84.7) \end{aligned}$ | 1.10 | 0.82 | 10,698. |
| $\begin{aligned} & 189.0 \\ & (33.5) \end{aligned}$ | $\begin{gathered} -104.7 \\ (40.6) \end{gathered}$ | $\begin{aligned} & -105.1 \\ & (39.8) \end{aligned}$ | 0.69 | 0.9 | 3,194.9 |
| $\begin{aligned} & -18.6 \\ & (31.1) \end{aligned}$ | $\begin{gathered} 12.8 \\ (31.7) \end{gathered}$ | $\begin{gathered} 34.6 \\ (30.4) \end{gathered}$ | 3.72* | 0.85 | 1,513. |
| $\begin{aligned} & 191.4 \\ & (31.5) \end{aligned}$ | $\begin{gathered} -104.7 \\ (40.3) \end{gathered}$ | $\begin{gathered} -126.3 \\ (39.4) \end{gathered}$ | 1.48 | 0.91 | 2,939. |
| $\begin{aligned} & -0.24 \\ & (0.42) \end{aligned}$ | $\begin{gathered} 0.91 \\ (0.40) \end{gathered}$ | $\begin{aligned} & -0.25 \\ & (0.39) \end{aligned}$ | 1.87 | 0.91 | . 0.24 |
| $\begin{aligned} & 179.1 \\ & (30.7) \end{aligned}$ | $\begin{aligned} & -84.7 \\ & (40.0) \end{aligned}$ | $\begin{array}{r} -112.0 \\ (37.9) \end{array}$ | 1.79 | 0.91 | 2,849. |
| $\begin{gathered} -9.8 \\ (29.5) \end{gathered}$ | $\begin{aligned} & -11.1 \\ & (30.0) \end{aligned}$ | $\begin{array}{r} 9.02 \\ (28.6) \end{array}$ | 0.42 | 0.78 | 1,384. |
| $\begin{aligned} & 171.4 \\ & (40.8)^{*} \end{aligned}$ | $\begin{gathered} 4.5 \\ (47.3) \end{gathered}$ | $\begin{aligned} & -75.4 \\ & (44.7) * * \end{aligned}$ | 0.41 | 0.9 | 3,936. |
| $\begin{aligned} & -0.17 \\ & (0.14) \end{aligned}$ | $\begin{gathered} 0.10 \\ (0.15) \end{gathered}$ | $\begin{gathered} 0.03 \\ (0.15) \end{gathered}$ | 0.95 | 0.84 | 0.04 |
| 133.2 | 19.79 | -52.5 | 4.3* | 0.57 | 354.3 |

Table B. 4 - continued


| $B$ (1) | $\beta$ (2) | $\beta$ (3) | $\beta$ (4) | $\beta$ (5) | $\beta$ (6) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & -2.5 \\ & (2.8) \end{aligned}$ | $\begin{gathered} 3.3 \\ (3.3) \end{gathered}$ | $\begin{gathered} -1.97 \\ (3.1) \end{gathered}$ | $\begin{aligned} & -0.07 \\ & (3.07) \end{aligned}$ | $\begin{aligned} & -2.15 \\ & (3.29) \end{aligned}$ | $\begin{gathered} 3.9 \\ (2.95) \end{gathered}$ |
| $\begin{gathered} 0.007 \\ (0.007) \end{gathered}$ | $\begin{gathered} 0.0002 \\ (0.007) \end{gathered}$ | $\begin{aligned} & -0.007 \\ & (0.006) \end{aligned}$ | $\begin{aligned} & -0.002 \\ & (0.006) \end{aligned}$ | $\begin{aligned} & -0.006 \\ & (0.007) . \end{aligned}$ | $\begin{aligned} & -0.007 \\ & (0.007) \end{aligned}$ |
| $\begin{gathered} 0.003 \\ (0.08) \end{gathered}$ | $\begin{aligned} & -0.17 \\ & (0.12) \end{aligned}$ | $\begin{gathered} 0.09 \\ (0.12) \end{gathered}$ | $\begin{aligned} & -0.02 \\ & (0.12) \end{aligned}$ | $\begin{gathered} 0.03 \\ (0.12) \end{gathered}$ | $\begin{gathered} 0.02 \\ (0.08) \end{gathered}$ |
| $\begin{aligned} & -0.13 \\ & (0.27) \end{aligned}$ | $\begin{gathered} 0.4 \\ (0.26) \end{gathered}$ | $\begin{aligned} & -0.03 \\ & (0.22) \end{aligned}$ | $\begin{aligned} & -0.11 \\ & (0.21) \end{aligned}$ | $\begin{aligned} & -0.03 \\ & (0.27) \end{aligned}$ | $\begin{aligned} & -0.11 \\ & (0.25) \end{aligned}$ |
| $\begin{gathered} 0.10 \\ (0.19) \end{gathered}$ | $\begin{aligned} & -0.27 \\ & (0.28) \end{aligned}$ | $\begin{aligned} & -0.14 \\ & (0.28) \end{aligned}$ | $\begin{gathered} 0.16 \\ (0.28) \end{gathered}$ | $\begin{gathered} 0.38 \\ (0.29) \end{gathered}$ | $\begin{aligned} & -0.14 \\ & (0.2) \end{aligned}$ |
| -0.012 | 0.1 | -0.08 | -0.05 | -0.02 | -0.18 |
| $\begin{gathered} 42.4 \\ (15.6)^{*} \end{gathered}$ | $\frac{-55.2}{(26.5)^{*}}$ | $\begin{gathered} 28.5 \\ (28.7) \end{gathered}$ | $\begin{gathered} -3.7 \\ (29.04) \end{gathered}$ | $\begin{aligned} & -16.2 \\ & (27.7) \end{aligned}$ | $\begin{gathered} 11.6 \\ (16.0) \end{gathered}$ |
| $\begin{gathered} 0.0005 \\ (0.001) \end{gathered}$ | $\begin{aligned} & -0.002 \\ & (0.001) \end{aligned}$ | $\begin{gathered} 0.0008 \\ (0.001) \end{gathered}$ | $\begin{aligned} & -0.002 \\ & (0.000) \end{aligned}$ | $\begin{aligned} & -0.00001 \\ & (0.001) \end{aligned}$ | $\begin{gathered} 0.001 \\ (0.000) \end{gathered}$ |
| $\begin{aligned} & -0.54 \\ & (0.23)^{*} \end{aligned}$ | $\begin{gathered} 0.51 \\ (0.34) \end{gathered}$ | $\begin{gathered} 0.15 \\ (0.34) \end{gathered}$ | $\begin{aligned} & -0.25 \\ & (0.33) \end{aligned}$ | $\begin{aligned} & -0.14 \\ & (0.37) \end{aligned}$ | $\begin{aligned} & -0.02 \\ & (0.28) \end{aligned}$ |
| $\begin{aligned} & -0.11 \\ & (0.009) \end{aligned}$ | $\begin{gathered} 0.05 \\ (0.09) \end{gathered}$ | $\begin{aligned} & -0.07 \\ & (0.07) \end{aligned}$ | $\begin{array}{r} 0.003 \\ (0.07) \end{array}$ | $\begin{aligned} & -0.04 \\ & (0.08) \end{aligned}$ | $\begin{aligned} & -0.05 \\ & (0.08) \end{aligned}$ |
| $\begin{aligned} & -0.02 \\ & (0.11) \end{aligned}$ | $\begin{gathered} 0.06 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.21 \\ & (0.14) \end{aligned}$ | $\begin{gathered} 0.19 \\ (0.15) \end{gathered}$ | $\begin{aligned} & -0.07 \\ & (0.14) \end{aligned}$ | $\begin{gathered} 0.05 \\ (0.11) \end{gathered}$ |
| $\begin{aligned} & -0.13 \\ & (0.15) \end{aligned}$ | $\begin{gathered} 0.34 \\ (0.15) \end{gathered}$ | $\begin{aligned} & -0.03 \\ & (0.08) \end{aligned}$ | $\begin{gathered} 0.15 \\ (0.07) \end{gathered}$ | $\begin{gathered} 0.29 \\ (0.15) \end{gathered}$ | $\begin{aligned} & -0.18 \\ & (0.16) \end{aligned}$ |
| $\begin{gathered} 15-18 \\ (14.5) \end{gathered}$ | $\begin{aligned} & -23.5 \\ & (16.6) \end{aligned}$ | $\begin{gathered} 09.9 \\ (15.1) \end{gathered}$ | $\begin{aligned} & -23.3 \\ & (15.1) \end{aligned}$ | $\begin{gathered} 18.9 \\ (15.3) \end{gathered}$ | $\begin{gathered} 8.9 \\ (12.2) \end{gathered}$ |

Table B. 4 - continued

| Mode1: $Y(t)=C+\sum_{s=1}^{\sum} \alpha(s) Y(t-s)+\sum_{k=1}^{\sum} \beta(k) X(t-k)+\gamma_{1}$ Fall $+\gamma_{2}$ Wint + |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Fall | Wint | Spri | $\mathrm{F}_{6,50}$ | $\mathrm{R}^{2}$ | MSE |
| $\begin{aligned} & 163.5 \\ & (39.1)^{*} \end{aligned}$ | $\begin{gathered} -7.5 \\ (44.2) \end{gathered}$ | $\begin{gathered} -73.7 \\ (43.1)^{*} \end{gathered}$ | 0.55 | 0.91 | 3,874 |
| $\begin{gathered} -0.54 \\ (2.2) \end{gathered}$ | $\begin{aligned} & -1.86 \\ & (2.3) \end{aligned}$ | $\begin{gathered} 0.72 \\ (2.2) \end{gathered}$ | 0.87 | 0.70 | 9.72 |
| $\begin{aligned} & 142.3 \\ & (40.5) \end{aligned}$ | $\begin{gathered} -6.89 \\ (42.5) \end{gathered}$ | $\begin{aligned} & -60.5 \\ & (42.2) \end{aligned}$ | 1.59 | 0.92 | 3,468. |
| $\begin{aligned} & 131.13 \\ & (77.6) \end{aligned}$ | $\begin{aligned} & 120.3 \\ & (81.1) \end{aligned}$ | $\begin{aligned} & -97.9 \\ & (79.3) \end{aligned}$ | 0.57 | 0.81 | 11,338. |
| $\begin{aligned} & 160.7 \\ & (40.1)^{*} \end{aligned}$ | $\begin{array}{r} 4.45 \\ (42.1) \end{array}$ | $\begin{aligned} & -71.8 \\ & (41.4)^{*} \end{aligned}$ | 1.63 | 0.92 | 3,453. |
| 21.9 | -0.6 | 6.3 | 0.94 | 0.81 | 1,966. |
| $\begin{aligned} & 157.7 \\ & (37.5)^{*} \end{aligned}$ | $\begin{array}{r} -9.36 \\ (42.7) \end{array}$ | $\begin{aligned} & -64.6 \\ & (42.0) * \end{aligned}$ | 1.55 | 0.91 | 3,482. |
| $\begin{gathered} 0.54 \\ (0.33) \end{gathered}$ | $\begin{aligned} & -0.08 \\ & (0.36) \end{aligned}$ | $\begin{aligned} & -0.05 \\ & (0.34) \end{aligned}$ | 2.23* | 0.91 | 0.23 |
| $\begin{aligned} & 163.4 \\ & (37.5) \end{aligned}$ | $\begin{gathered} -6.8 \\ (43.4) \end{gathered}$ | $\begin{aligned} & -66.6 \\ & (42.8) \end{aligned}$ | 1.39 | 0.91 | 3.539. |
| $\begin{aligned} & -22.3 \\ & (25.3) \end{aligned}$ | $\begin{gathered} 31.2 \\ (27.5) \end{gathered}$ | $\begin{gathered} -6.5 \\ (26.4) \end{gathered}$ | 0.76 | 0.78 | 1,332.5 |
| $\begin{aligned} & 157.7 \\ & (39.4) \end{aligned}$ | $\begin{array}{r} 1.49 \\ (43.7) \end{array}$ | $\begin{aligned} & -54.4 \\ & (42.1) \end{aligned}$ | $1.88 * *$ | 0.92 | 3.369.2 |
|  |  |  | 3.5* | 0.63 | 4,860. |
| $\begin{aligned} & 187.5 \\ & (71.6) \end{aligned}$ | $\begin{gathered} -126.5 \\ (91.3) \end{gathered}$ | $\begin{aligned} & -112.5 \\ & (74.9) \end{aligned}$ | 2.11** | 0.92 | 3,294. |

Table B. 4 - continued

| Model: <br> Y | $\begin{aligned} & Y(t)= C+\sum_{s=1}^{4} \alpha(s) Y(t-s)+ \\ & \gamma_{3} S p r i \end{aligned} e_{t}$ |  | $\sum_{k=1}^{6} \beta(k) X(t-k)+\gamma_{1} \text { Fall }+\gamma_{2} \text { Wint }+$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | X | $\begin{gathered} c \\ (\mathrm{~s} \cdot \mathrm{e}) \\ \hline \end{gathered}$ | $\alpha$ (1) | $\alpha$ (2) | $\alpha(3)$ | $\alpha(4)$ |
| HPAU | SBC | $\begin{aligned} & -0.002 \\ & (0.07) \end{aligned}$ | $\begin{gathered} 0.79 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.06 \\ & (0.15) \end{aligned}$ | $\begin{gathered} 0.16 \\ (0.15) \end{gathered}$ | $\begin{aligned} & -0.14 \\ & (0.12) \end{aligned}$ |
| SC | CORPF | $\begin{aligned} & -4.36 \\ & (1.88) \end{aligned}$ | $\begin{gathered} 0.74 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.26 \\ & (0.17) \end{aligned}$ | $\begin{aligned} & -0.22 \\ & (0.17) \end{aligned}$ | $\begin{gathered} 0.06 \\ (0.15) \end{gathered}$ |
| CORPF | SC | $\begin{aligned} & -0.02 \\ & (0.03) \end{aligned}$ | $\begin{gathered} 0.86 \\ (0.14) \end{gathered}$ | $\begin{gathered} 0.16 \\ (0.19) \end{gathered}$ | $\begin{gathered} 0.12 \\ (0.19) \end{gathered}$ | $\begin{aligned} & -0.28 \\ & (0.13) \end{aligned}$ |
| SC | SBX | $\begin{aligned} & -3.43 \\ & (2.02) \end{aligned}$ | $\begin{gathered} 1.11 \\ (0.18) \end{gathered}$ | $\begin{aligned} & -0.43 \\ & (0.25) \end{aligned}$ | $\begin{aligned} & -0.24 \\ & (0.25) \end{aligned}$ | $\begin{array}{r} 0.09 \\ (0.2) \end{array}$ |
| SBX | SC | $\begin{aligned} & -4.28 \\ & (2.89) \end{aligned}$ | $\begin{aligned} & -0.11 \\ & (0.17) \end{aligned}$ | $\begin{aligned} & -0.16 \\ & (0.19) \end{aligned}$ | $\begin{gathered} 0.13 \\ (0.18) \end{gathered}$ | $\begin{gathered} 0.42 \\ (0.18) \end{gathered}$ |
| SC | HPAU | $\begin{aligned} & -3.5 \\ & (1.77) \end{aligned}$ | $\begin{gathered} 0.64 \\ (0.15) \end{gathered}$ | $\begin{aligned} & -0.27 \\ & (0.17) \end{aligned}$ | $\begin{aligned} & -0.27 \\ & (0.17) \end{aligned}$ | $\begin{gathered} 0.19 \\ (0.14) \end{gathered}$ |
| . HPAU | SC | $\begin{aligned} & -0.0003 \\ & (0.09) \end{aligned}$ | $\begin{gathered} 0.79 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.08 \\ & (0.17) \end{aligned}$ | $\begin{gathered} 0.11 \\ (0.17) \end{gathered}$ | $\begin{aligned} & -0.07 \\ & (0.14) \end{aligned}$ |
| SC | COOP | $\begin{aligned} & -4.11 \\ & (1.91) \end{aligned}$ | $\begin{gathered} 0.69 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.21 \\ & (0.18) \end{aligned}$ | $\begin{aligned} & -0.22 \\ & (0.19) \end{aligned}$ | $\begin{gathered} 0.07 \\ (0.16) \end{gathered}$ |
| COOP | SC | $\begin{aligned} & -0.20 \\ & (0.43) \end{aligned}$ | $\begin{gathered} 0.63 \\ (0.13) \end{gathered}$ | $\begin{gathered} 0.39 \\ (0.14) \end{gathered}$ | $\begin{gathered} 0.19 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.49 \\ & (0.12) \end{aligned}$ |
| SC | SOH | $\begin{aligned} & -2.9 \\ & (1.8) \end{aligned}$ | $\begin{aligned} & 0.6 \\ & (0.17) \end{aligned}$ | $\begin{array}{r} 0.006 \\ (0.21) \end{array}$ | $\begin{aligned} & -0.31 \\ & (0.2) \end{aligned}$ | $\begin{gathered} 0.32 \\ (0.18) \end{gathered}$ |
| SOH | SC | $\begin{aligned} & -13.7 \\ & (14.6) \end{aligned}$ | $\begin{gathered} 1.22 \\ (0.17) \end{gathered}$ | $\begin{aligned} & -0.39 \\ & (0.26) \end{aligned}$ | $\begin{gathered} 0.22 \\ (0.27) \end{gathered}$ | $\begin{aligned} & -0.24 \\ & (0.18) \end{aligned}$ |
| SC | SMH | $\begin{aligned} & -3.06 \\ & (1.72) \end{aligned}$ | $\begin{gathered} 0.62 \\ (0.15) \end{gathered}$ | $\begin{aligned} & -0.20 \\ & (0.17) \end{aligned}$ | $\begin{aligned} & -0.20 \\ & (0.17) \end{aligned}$ | $\begin{gathered} 0.16 \\ (0.15) \end{gathered}$ |
| SMH | SC | $\begin{gathered} -1.19 \\ (6.6) \end{gathered}$ | $\begin{gathered} 1.07 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.31 \\ & (0.22) \end{aligned}$ | $\begin{gathered} 0.38 \\ (0.23) \end{gathered}$ | $\begin{aligned} & -0.36 \\ & (0.16) \end{aligned}$ |


| $\beta$ (1) | $\beta$ (2) | $\beta$ (3) | $\beta$ (4) | $\beta$ (5) | $\beta$ (6) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 0.002 \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.001 \\ (0.001) \end{gathered}$ | $\begin{aligned} & -0.003 \\ & (0.001) \end{aligned}$ | $\begin{gathered} 0.001 \\ (0.001) \end{gathered}$ | $\begin{array}{r} 0.0007 \\ (0.001) \end{array}$ | $\begin{gathered} 0.002 \\ (0.001) \end{gathered}$ |
| $\begin{gathered} -15.35 \\ (9.03) \end{gathered}$ | $\begin{gathered} 18.8 \\ (12.4) \end{gathered}$ | $\begin{gathered} -6.13 \\ (12.4) \end{gathered}$ | $\begin{gathered} -0.84 \\ (12.5) \end{gathered}$ | $\begin{aligned} & -13.85 \\ & (13.5) \end{aligned}$ | $\begin{gathered} 8.72 \\ (9.7) \end{gathered}$ |
| $\begin{gathered} 0.001 \\ (0.002) \end{gathered}$ | $\begin{aligned} & -0.002 \\ & (0.003) \end{aligned}$ | $\begin{aligned} & -0.0001 \\ & (0.003) \end{aligned}$ | $\begin{gathered} 0.0003 \\ (0.003) \end{gathered}$ | $\begin{aligned} & -0.007 \\ & (0.003) \end{aligned}$ | $\begin{gathered} 0.007 \\ (0.003) \end{gathered}$ |
| $\begin{aligned} & -0.32 \\ & (0.13) \end{aligned}$ | $\begin{gathered} 0.09 \\ (0.14) \end{gathered}$ | $\begin{gathered} 0.13 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.01 \\ & (0.12) \end{aligned}$ | $\begin{gathered} 0.14 \\ (0.11) \end{gathered}$ | $\begin{aligned} & -0.15 \\ & (0.10) \end{aligned}$ |
| $\begin{gathered} 0.36 \\ (0.26) \end{gathered}$ | $\begin{gathered} 0.34 \\ (0.35) \end{gathered}$ | $\begin{aligned} & -0.75 \\ & (0.36) \end{aligned}$ | $\begin{gathered} 0.04 \\ (0.38) \end{gathered}$ | $\begin{gathered} 0.02 \\ (0.33) \end{gathered}$ | $\begin{gathered} 0.06 \\ (0.24) \end{gathered}$ |
| $\begin{array}{r} 3.56 \\ (2.8) \end{array}$ | $\begin{gathered} 1.53 \\ (3.4) \end{gathered}$ | $\begin{gathered} 3.51 \\ (3.2) \end{gathered}$ | $\begin{aligned} & -5.76 \\ & (3.2) \end{aligned}$ | $\begin{aligned} & -1.67 \\ & (3.2) \end{aligned}$ | $\begin{gathered} 0.72 \\ (2.6) \end{gathered}$ |
| $\begin{aligned} & 0.01 \\ & (0.007) \end{aligned}$ | $\begin{aligned} & -0.01 \\ & (0.008) \end{aligned}$ | $\begin{aligned} & -0.003 \\ & (0.009) \end{aligned}$ | $\begin{gathered} 0.01 \\ (0.009) \end{gathered}$ | $\begin{aligned} & -0.01 \\ & (0.01) \end{aligned}$ | $\begin{gathered} 0.008 \\ (0.008) \end{gathered}$ |
| $\begin{aligned} & -0.54 \\ & (0.64) \end{aligned}$ | $\begin{aligned} & -0.04 \\ & (0.74) \end{aligned}$ | $\begin{gathered} 0.06 \\ (0.69) \end{gathered}$ | $\begin{gathered} 0.05 \\ (0.65) \end{gathered}$ | $\begin{aligned} & -0.13 \\ & (0.72) \end{aligned}$ | $\begin{aligned} & -0.22 \\ & (0.63) \end{aligned}$ |
| $\begin{gathered} 0.07 \\ (0.03) \end{gathered}$ | $\begin{aligned} & -0.05 \\ & (0.04) \end{aligned}$ | $\begin{aligned} & -0.07 \\ & (0.04) \end{aligned}$ | $\begin{gathered} 0.06 \\ (0.04) \end{gathered}$ | $\begin{aligned} & -0.08 \\ & (0.05) \end{aligned}$ | $\begin{aligned} & 0.02 \\ & 0.04 \end{aligned}$ |
| $\begin{gathered} 0.02 \\ (0.02) \end{gathered}$ | $\begin{aligned} & -0.02 \\ & (0.03) \end{aligned}$ | $\begin{gathered} 0.02 \\ (0.03) \end{gathered}$ | $\begin{aligned} & -0.016 \\ & (0.03) \end{aligned}$ | $\begin{gathered} 0.02 \\ (0.03) \end{gathered}$ | $\begin{aligned} & -0.004 \\ & (0.02) \end{aligned}$ |
| $\begin{aligned} & -0.75 \\ & (1.4) \end{aligned}$ | $\begin{aligned} & 2.9 \\ & (1.73) \end{aligned}$ | $\begin{aligned} & -4.79 \\ & (1.78) \end{aligned}$ | $\begin{gathered} 1.58 \\ (1.75) \end{gathered}$ | $\begin{aligned} & -0.34 \\ & (1.92) \end{aligned}$ | $\begin{gathered} 1.22 \\ (1.47) \end{gathered}$ |
| $\begin{gathered} 0.02 \\ (0.04) \end{gathered}$ | $\begin{aligned} & -0.11 \\ & (0.06) \end{aligned}$ | $\begin{gathered} 0.06 \\ (0.06) \end{gathered}$ | $\begin{aligned} & -0.07 \\ & (0.06) \end{aligned}$ | $\begin{gathered} 0.06 \\ (0.06) \end{gathered}$ | $\begin{aligned} & -0.003 \\ & (0.04) \end{aligned}$ |
| $\begin{gathered} 0.16 \\ (0.54) \end{gathered}$ | $\begin{gathered} 0.15 \\ (0.62) \end{gathered}$ | $\begin{aligned} & -0.84 \\ & (0.62) \end{aligned}$ | $\begin{gathered} 0.08 \\ (0.64) \end{gathered}$ | $\begin{aligned} & -0.81 \\ & (0.76) \end{aligned}$ | $\begin{gathered} 0.60 \\ (0.61) \end{gathered}$ |

Table B. 4 - continued

|  | $\begin{aligned} Y(t)= & c+\sum_{s=1}^{4} \alpha(s) Y(t-s)+ \\ & \gamma_{3} S p r i+e_{t} \end{aligned}$ |  | $\sum_{k=1}^{6} \beta(k) X(t-k)+\gamma_{1} \text { Fall }+\gamma_{2} \text { Wint }+$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Wint | Spri | $\mathrm{F}_{6,50}$ | $\mathrm{R}^{2}$ | MSE |
| $\begin{gathered} 4.63 \\ (0.72) \end{gathered}$ | $\begin{aligned} & -1.19 \\ & (0.86) \end{aligned}$ | $\begin{aligned} & -5.05 \\ & (0.71) \end{aligned}$ | 4.6* | 0.96 | 0.28 |
| $\begin{gathered} 9.64 \\ (3.5) \end{gathered}$ | $\begin{array}{r} 5.23 \\ (3.6) \end{array}$ | $\begin{aligned} & -3.58 \\ & (3.6) \end{aligned}$ | 1.32 | 0.60 | 152.2 |
| $\begin{aligned} & -0.15 \\ & (0.06) \end{aligned}$ | $\begin{gathered} 0.04 \\ (0.06) \end{gathered}$ | $\begin{gathered} 0.07 \\ (0.06) \end{gathered}$ | 2.12** | 0.85 | 0.03 |
| $\begin{gathered} 5.7 \\ (4.62) \end{gathered}$ | $\begin{gathered} 7.8 \\ (4.29) \end{gathered}$ | $\begin{aligned} & -2.89 \\ & (4.2) \end{aligned}$ | 1.69 | 0.62 | 146.5 |
| $\begin{aligned} & 12.5 \\ & (6.8) \end{aligned}$ | $\begin{gathered} 7.8 \\ (6.5) \end{gathered}$ | $\begin{array}{r} 3.01 \\ (6.9) \end{array}$ | 2.26** | 0.62 | 309.8 |
| $\begin{gathered} 29.1 \\ (13.5) \end{gathered}$ | $\begin{gathered} 17.7 \\ (15.1) \end{gathered}$ | $\begin{aligned} & -25.1 \\ & (13.8) \end{aligned}$ | 1.66 | 0.61 | 146.9 |
| $\begin{array}{r} 3.59 \\ (0.6) \end{array}$ | $\begin{gathered} -1.28 \\ (0.7) \end{gathered}$ | $\begin{aligned} & -3.3 \\ & (0.63) \end{aligned}$ | 1.18 | 0.94 | 0.38 |
| $\begin{gathered} 7.57 \\ (3.45) \end{gathered}$ | $\begin{aligned} & (8.04) \\ & (3.36) \end{aligned}$ | $\begin{aligned} & -4.7 \\ & (3.5) \end{aligned}$ | 0.55 | 0.57 | 165.4 |
| $\begin{array}{r} -0.28 \\ 0.86 \end{array}$ | $\begin{aligned} & -0.47 \\ & (0.81) \end{aligned}$ | $\begin{gathered} 0.05 \\ (0.87) \end{gathered}$ | 2.89* | 0.76 | 7.97 |
| $\begin{aligned} & 11.82 \\ & (4.5) \end{aligned}$ | $\begin{gathered} 2.81 \\ (4.23) \end{gathered}$ | $\begin{aligned} & -3.94 \\ & (3.88) \end{aligned}$ | 1.25 | 0.6 | 153.4 |
| $\begin{aligned} & 132.89 \\ & (34.1) \end{aligned}$ | $\begin{array}{r} 40.85 \\ (36.9) \end{array}$ | $\begin{aligned} & -27.55 \\ & (37.6) \end{aligned}$ | 1.75 | 0.83 | 10,008. |
| $\begin{gathered} 9.45 \\ (3.96) \end{gathered}$ | $\begin{gathered} 5.15 \\ (3.49) \end{gathered}$ | $\begin{aligned} & -3.04 \\ & (3.49) \end{aligned}$ | 2.13** | 0.63 | 140.5 |
| $\begin{array}{r} 16.48 \\ (13.8) \end{array}$ | $\begin{gathered} 13.8 \\ (14.1) \end{gathered}$ | $\begin{gathered} 15.3 \\ (14.5) \end{gathered}$ | 1.25 | 0.81 | 1,903.7 |

Table B. 4 - continued

| Model: | $\begin{aligned} & Y(t)= C+\sum_{s=1}^{4} \alpha(s) Y(t-s)+\sum_{k=1}^{6} \beta(k) X(t-k)+\gamma_{1} \text { Fall }+\gamma_{2} \text { Wint }+ \\ & \gamma_{3} \text { Spri }+e_{t} \end{aligned}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | X | $\begin{gathered} c \\ (\mathrm{~s} \cdot \mathrm{e}) \end{gathered}$ | $\alpha(1)$ | $\alpha$ (2) | $\alpha$ (3) | $\alpha(4)$ |
| SC | TB | $\begin{aligned} & -4.4 \\ & (6.8) \end{aligned}$ | $\begin{gathered} 0.77 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.22 \\ & (0.18) \end{aligned}$ | $\begin{aligned} & -0.24 \\ & (0.18) \end{aligned}$ | $\begin{gathered} 0.16 \\ (0.15) \end{gathered}$ |
| TB | Sc | $\begin{gathered} 0.65 \\ (0.25) \end{gathered}$ | $\begin{gathered} 1.21 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.45 \\ & (0.21) \end{aligned}$ | $\begin{gathered} 0.42 \\ (0.21) \end{gathered}$ | $\begin{aligned} & -0.3 \\ & (0.13) \end{aligned}$ |
| SC | FIMP | $\begin{aligned} & -3.52 \\ & (1.56) \end{aligned}$ | $\begin{gathered} 0.73 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.17 \\ & (0.17) \end{aligned}$ | $\begin{aligned} & -0.25 \\ & (0.17) \end{aligned}$ | $\begin{gathered} 0.13 \\ (0.13) \end{gathered}$ |
| FIMP | SC | $\begin{aligned} & -2.98 \\ & (5.2) \end{aligned}$ | $\begin{gathered} 0.98 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.08 \\ & (0.2) \end{aligned}$ | $\begin{gathered} 0.02 \\ (0.22) \end{gathered}$ | $\begin{aligned} & -0.27 \\ & (0.14) \end{aligned}$ |
| PS | CORPF | $\begin{aligned} & -0.05 \\ & (0.1) \end{aligned}$ | $\begin{gathered} 0.73 \\ (0.17) \end{gathered}$ | $\begin{gathered} 0.11 \\ (0.24) \end{gathered}$ | $\begin{aligned} & -0.43 \\ & (0.25) \end{aligned}$ | $\begin{gathered} 0.35 \\ (0.18) \end{gathered}$ |
| CORPF | PS | $\begin{aligned} & -0.01 \\ & (0.02) \end{aligned}$ | $\begin{gathered} 1.39 \\ (0.17) \end{gathered}$ | $\begin{aligned} & -0.99 \\ & (0.25) \end{aligned}$ | $\begin{gathered} 0.86 \\ (0.26) \end{gathered}$ | $\begin{aligned} & -0.48 \\ & (0.17) \end{aligned}$ |
| PS | SBX | $\begin{gathered} 0.04 \\ (0.13) \end{gathered}$ | $\begin{gathered} 0.75 \\ (0.14) \end{gathered}$ | $\begin{gathered} 0.02 \\ (0.18) \end{gathered}$ | $\begin{aligned} & \cdot 0.02 \\ & (0.19) \end{aligned}$ | $\begin{gathered} 0.02 \\ (0.15) \end{gathered}$ |
| SBX | PS | $\begin{aligned} & -4.86 \\ & (3.2) \end{aligned}$ | $\begin{gathered} 0.09 \\ (0.13) \end{gathered}$ | $\begin{gathered} 0.04 \\ (0.16) \end{gathered}$ | $\begin{aligned} & -0.11 \\ & (0.15) \end{aligned}$ | $\begin{gathered} 0.36 \\ (0.15) \end{gathered}$ |
| PS | HPAU | $\begin{gathered} -0.01 \\ (0.1) \end{gathered}$ | $\begin{gathered} 0.82 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.18 \\ & (0.18) \end{aligned}$ | $\begin{array}{r} 0.058 \\ (0.18) \end{array}$ | $\begin{gathered} 0.12 \\ (0.14) \end{gathered}$ |
| HPAU | PS | $\begin{aligned} & -0.11 \\ & (0.08) \end{aligned}$ | $\begin{gathered} 0.72 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.14 \\ & (0.16) \end{aligned}$ | $\begin{gathered} 0.16 \\ (0.16) \end{gathered}$ | $\begin{aligned} & -0.04 \\ & (0.13) \end{aligned}$ |
| PS | COOP | $\begin{aligned} & -0.03 \\ & (0.1) \end{aligned}$ | $\begin{gathered} 1.08 \\ (0.17) \end{gathered}$ | $\begin{aligned} & -0.42 \\ & (0.23) \end{aligned}$ | $\begin{gathered} 0.002 \\ (0.23) \end{gathered}$ | $\begin{gathered} 0.37 \\ (0.19) \end{gathered}$ |
| COOP | PS | $\begin{gathered} 0.23 \\ (0.35) \end{gathered}$ | $\begin{gathered} 0.44 \\ (0.17) \end{gathered}$ | $\begin{gathered} 0.27 \\ (0.2) \end{gathered}$ | $\begin{aligned} & -0.004 \\ & (0.18) \end{aligned}$ | $\begin{aligned} & -0.41 \\ & (0.13) \end{aligned}$ |
| PS | SOH | $\begin{gathered} 0.02 \\ (0.09) \end{gathered}$ | $\begin{gathered} 0.92 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.18 \\ & (0.10) \end{aligned}$ | $\begin{aligned} & -0.04 \\ & (0.19) \end{aligned}$ | $\begin{gathered} 0.13 \\ (0.15) \end{gathered}$ |


| $\beta(1)$ | $\beta(2)$ | $\beta(3)$ | $\beta(4)$ | $\beta(5)$ | $\beta(6)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| -1.15 | 3.38 | -1.75 | -0.61 | -0.17 | 0.55 |
| $(3.68)$ | $(5.8)$ | $(6.01)$ | $(5.9)$ | $(5.67)$ | $(3.42)$ |
| 0.0001 | -0.002 | -0.004 | -0.008 | 0.008 | 0.003 |
| $(0.005)$ | $(0.007)$ | $(0.007)$ | $(0.007)$ | $(0.008)$ | $(0.007)$ |
| -0.18 | 0.13 | 0.06 | -0.02 | -0.04 | -0.05 |
| $(0.04)$ | $(0.07)$ | $(0.06)$ | $(0.07)$ | $(0.07)$ | $(0.05)$ |
| 0.08 | -0.35 | 0.06 | 0.76 | -1.81 | 0.92 |
| $(0.43)$ | $(0.57)$ | $(0.55)$ | $(0.51)$ | $(0.61)$ | $(0.49)$ |
| 1.18 | -2.32 | 2.23 | -0.7 | -0.54 | 0.14 |
| $(0.76)$ | $(1.08)$ | $(0.99)$ | $(0.84)$ | $(0.89)$ | $(0.66)$ |
| -0.09 | 0.22 | -0.24 | 0.22 | -0.05 | 0.002 |
| $(0.03)$ | $(0.05)$ | $(0.06)$ | $(0.05)$ | $(0.05)$ | $(0.04)$ |
| 0.02 | 0.004 | -0.002 | 0.001 | -0.01 | 0.003 |
| $(0.006)$ | $(0.007)$ | $(0.006)$ | $(0.01)$ | $(0.01)$ | $(0.01)$ |
| -6.63 | 3.31 | -1.77 | 3.34 | 0.18 | -2.16 |
| $(3.44)$ | $(4.47)$ | $(4.43)$ | $(4.34)$ | $(4.44)$ | $(3.74)$ |
| 0.08 | -0.03 | 0.19 | 0.05 | -0.06 | -0.11 |
| $(0.18)$ | $(0.22)$ | $(0.22)$ | $(0.22)$ | $(0.22)$ | $(0.17)$ |
| 0.007 | -0.14 | 0.11 | -0.06 | -0.09 | -0.005 |
| $(0.10)$ | $(0.13)$ | $(0.14)$ | $(0.14)$ | $(0.14)$ | $(0.12)$ |
| -0.11 | 0.10 | 0.004 | -0.06 | -0.05 | 0.04 |
| $(0.05)$ | $(0.05)$ | $(0.05)$ | $(0.04)$ | $(0.04)$ | $(0.04)$ |
| 0.79 | -1.09 | 2.13 | -0.31 | 1.57 | -0.13 |
| $(0.6)$ | $(0.8)$ | $(0.79)$ | $(0.85)$ | $(0.81)$ | $(0.75)$ |
| 0.003 | -0.003 | -0.001 | 0.003 | -0.003 | 0.001 |
| $(0.001)$ | $(0.001)$ | $(0.002)$ | $(0.002)$ | $(0.002)$ | $(0.001)$ |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

Table B. 4 - continued


Table B. 4 - continued


| $\beta$ (1) | $\beta$ (2) | $\beta$ (3) | $\beta$ (4) | $\beta$ (5) | $\beta(6)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & -33.3 \\ & (19.3) \end{aligned}$ | $\begin{gathered} 15.0 \\ (25.5) \end{gathered}$ | $\begin{aligned} & -20.3 \\ & (24.4) \end{aligned}$ | $\begin{aligned} & 54.5 \\ & (22.02) \end{aligned}$ | $\begin{aligned} & -31.8 \\ & (24.04) \end{aligned}$ | $\begin{gathered} 16.46 \\ (19.9) \end{gathered}$ |
| $\begin{gathered} 0.003 \\ (0.003) \end{gathered}$ | $\begin{aligned} & -0.001 \\ & (0.004) \end{aligned}$ | $\begin{gathered} 0.001 \\ (0.004) \end{gathered}$ | $\begin{aligned} & -0.002 \\ & (0.004) \end{aligned}$ | $\begin{aligned} & -0.002 \\ & (0.004) \end{aligned}$ | $\begin{gathered} 0.002 \\ (0.003) \end{gathered}$ |
| $\begin{gathered} 8.23 \\ (7.2) \end{gathered}$ | $\begin{aligned} & -7.35 \\ & (9.4) \end{aligned}$ | $\begin{aligned} & 14.9 \\ & (9.44) \end{aligned}$ | $\begin{aligned} & -5.8 \\ & (9.6) \end{aligned}$ | $\begin{aligned} & 12.6 \\ & (9.7) \end{aligned}$ | $\begin{aligned} & -1.6 \\ & (8.8) \end{aligned}$ |
| $\begin{aligned} & -0.009 \\ & (0.23) \end{aligned}$ | $\begin{gathered} 0.19 \\ (0.38) \end{gathered}$ | $\begin{aligned} & -0.2 \\ & (0.4) \end{aligned}$ | $\begin{gathered} 0.07 \\ (0.4) \end{gathered}$ | $\begin{aligned} & -0.16 \\ & (0.37) \end{aligned}$ | $\begin{gathered} 0.06 \\ (0.22) \end{gathered}$ |
| $\begin{gathered} 0.05 \\ (0.09) \end{gathered}$ | $\begin{aligned} & -0.13 \\ & (0.11) \end{aligned}$ | $\begin{gathered} 0.18 \\ (0.11) \end{gathered}$ | $\begin{aligned} & -0.1 \\ & (0.11) \end{aligned}$ | $\begin{gathered} 0.17 \\ (0.12) \end{gathered}$ | $\begin{aligned} & -0.23 \\ & (0.09) \end{aligned}$ |
| $\begin{gathered} 0.02 \\ (0.004) \end{gathered}$ | $\begin{aligned} & -0.01 \\ & (0.005) \end{aligned}$ | $\begin{gathered} 0.004 \\ (0.004) \end{gathered}$ | $\begin{aligned} & -0.001 \\ & (0.004) \end{aligned}$ | $\begin{aligned} & -0.001 \\ & (0.004) \end{aligned}$ | $\begin{gathered} 0.007 \\ (0.004) \end{gathered}$ |
| $\begin{gathered} -17.92 \\ (8.18) \end{gathered}$ | $\begin{aligned} & 26.4 \\ & (9.6) \end{aligned}$ | $\begin{aligned} & -33.5 \\ & (10.3) \end{aligned}$ | $\begin{gathered} 23.6 \\ (10.4) \end{gathered}$ | $\begin{aligned} & 06.5 \\ & (8.67) \end{aligned}$ | $\begin{gathered} 8.9 \\ (7.8) \end{gathered}$ |
| $\begin{aligned} & -28.3 \\ & (21.8) \end{aligned}$ | $\begin{aligned} & 2.8 \\ & (32.8) \end{aligned}$ | $\begin{gathered} 52.7 \\ (32.0) \end{gathered}$ | $\begin{aligned} & -40.5 \\ & (26.7) \end{aligned}$ | $\begin{gathered} -8.0 \\ (25.5) \end{gathered}$ | $\begin{gathered} 15.5 \\ (19.3) \end{gathered}$ |
| $\begin{aligned} & -0.0005 \\ & (0.001) \end{aligned}$ | $\begin{gathered} 0.003 \\ (0.002) \end{gathered}$ | $\begin{aligned} & -0.003 \\ & (0.002) \end{aligned}$ | $\begin{gathered} 0.002 \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.006 \\ (0.002) \end{gathered}$ | $\begin{aligned} & -0.00004 \\ & (0.001) \end{aligned}$ |
| $\begin{gathered} 0.17 \\ (0.2) \end{gathered}$ | $\begin{aligned} & -0.09 \\ & (0.2) \end{aligned}$ | $\begin{aligned} & -0.05 \\ & (0.19) \end{aligned}$ | $\begin{aligned} & -0.09 \\ & (0.18) \end{aligned}$ | $\begin{aligned} & -0.06 \\ & (0.2) \end{aligned}$ | $\begin{gathered} 0.03 \\ (0.2) \end{gathered}$ |
| $\begin{aligned} & -0.14 \\ & (0.11) \end{aligned}$ | $\begin{gathered} 0.08 \\ (0.16) \end{gathered}$ | $\begin{aligned} & -0.12 \\ & (0.15) \end{aligned}$ | $\begin{gathered} 0.21 \\ (0.16) \end{gathered}$ | $\begin{gathered} 0.06 \\ (0.19) \end{gathered}$ | $\begin{aligned} & -0.23 \\ & (0.15) \end{aligned}$ |
| $\begin{gathered} 0.13 \\ (5.4) \end{gathered}$ | $\begin{aligned} & -0.60 \\ & (6.6) \end{aligned}$ | $\begin{gathered} 5.9 \\ (6.5) \end{gathered}$ | $\begin{gathered} -1.39 \\ (6.5) \end{gathered}$ | $\begin{aligned} & -3.88 \\ & (6.5) \end{aligned}$ | $\begin{gathered} 0.98 \\ (5.1) \end{gathered}$ |
| 0.001 | -0.003 | -0.00003 | 0.003 | -0.002 | -0.007 |

Table B. 4 - continued

| Model: <br> Fall | $\begin{aligned} Y(t)= & C+\underset{s=1}{\Sigma} \alpha(s) Y(t-s)+ \\ & \gamma_{3} \operatorname{Spri}+e_{t} \end{aligned}$ |  | $\sum_{k=1}^{6} \beta(k) X(t-k)+\gamma_{1} \text { Fall }+\gamma_{2} \text { Wint }+$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Wint | Spri | $\mathrm{F}_{6,50}$ | $\mathrm{R}^{2}$ | MSE |
| $\begin{aligned} & 129.06 \\ & (29.1) \end{aligned}$ | $\begin{gathered} 23.4 \\ (33.5) \end{gathered}$ | $\begin{aligned} & -34.7 \\ & (30.6) \end{aligned}$ | 1.77 | 0.83 | 9,983.6 |
| $\begin{aligned} & -0.36 \\ & (0.26) \end{aligned}$ | $\begin{gathered} 0.09 \\ (0.23) \end{gathered}$ | $\begin{gathered} 0.21 \\ (0.23) \end{gathered}$ | 0.58 | 0.60 | 0.70 |
| $\begin{gathered} 14.6 \\ (11.3) \end{gathered}$ | $\begin{gathered} 11.2 \\ (11.8) \end{gathered}$ | $\begin{gathered} 10.6 \\ (11.5) \end{gathered}$ | 1.59 | 0.82 | 1,837.2 |
| $\begin{aligned} & -0.3 \\ & (0.21) \end{aligned}$ | $\begin{gathered} 0.04 \\ (0.2) \end{gathered}$ | $\begin{gathered} 0.15 \\ (0.21) \end{gathered}$ | 0.26 | 0.59 | 0.73 |
| $\begin{aligned} & -0.10 \\ & (0.12) \end{aligned}$ | $\begin{aligned} & -0.03 \\ & (0.13) \end{aligned}$ | $\begin{aligned} & -0.19 \\ & (0.12) \end{aligned}$ | 1.49 | 0.9 | 0.25 |
| $\begin{aligned} & -0.14 \\ & (0.17) \end{aligned}$ | $\begin{aligned} & -0.07 \\ & (0.17) \end{aligned}$ | $\begin{gathered} 0.14 \\ (0.17) \end{gathered}$ | 3.75* | 0.71 | 0.52 |
| $\begin{array}{r} 8.58 \\ (7.9) \end{array}$ | $\begin{aligned} & -2.5 \\ & (8.4) \end{aligned}$ | $\begin{aligned} & 6.4 \\ & (8.03) \end{aligned}$ | 2.5* | 0.82 | 1,118.6 |
| $\begin{aligned} & -5.5 \\ & (5.7) \end{aligned}$ | $\begin{gathered} 3.2 \\ (5.99) \end{gathered}$ | $\begin{array}{r} 1.66 \\ (5.6) \end{array}$ | 1.118 | 0.66 | 563.2 |
| $\begin{aligned} & -0.1 \\ & (0.04) \end{aligned}$ | $\begin{gathered} 0.06 \\ (0.05) \end{gathered}$ | $\begin{aligned} & -0.01 \\ & (0.04) \end{aligned}$ | 1.66 | 0.85 | 0.03 |
| $\begin{gathered} -1.27 \\ (8.3) \end{gathered}$ | $\begin{gathered} 2.18 \\ (8.76) \end{gathered}$ | $\begin{gathered} 3.04 \\ (8.4) \end{gathered}$ | 0.27 | 0.63 | 618.9 |
| $\begin{aligned} & 12.06 \\ & (6.62) \end{aligned}$ | $\begin{gathered} 5.42 \\ (6.22) \end{gathered}$ | $\begin{array}{r} 3.49 \\ (6.2) \end{array}$ | 1.51 | 0.59 | 333.5 |
| $\begin{aligned} & -10.05 \\ & (26.3) \end{aligned}$ | $\begin{gathered} 32.4 \\ (29.5) \end{gathered}$ | $\begin{array}{r} 7.46 \\ (26.7) \end{array}$ | 0.29 | 0.63 | 617.2 |
| 3.13 | -0.69 | -2.9 | 1.95** | 0.95 | 0.36 |

Table B. 4 - continued

| Model: <br> Y | $\begin{aligned} Y(t)= & C+\sum_{s=1}^{4} \alpha(s) Y(t-s)+\sum_{k=1}^{6} \beta(k) X(t-k)+\gamma_{1} \text { Fall }+\gamma_{2} \text { Wint }+ \\ & \gamma_{3} S p r i+e_{t} \end{aligned}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | X | $\begin{gathered} c \\ (\mathrm{~s} \cdot \mathrm{e}) \end{gathered}$ | $\alpha$ (1) | $\alpha$ (2) | $\alpha$ (3) | $\alpha(4)$ |
| PM | COOP | $\begin{aligned} & -1.7 \\ & (2.86) \end{aligned}$ | $\begin{gathered} 1.07 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.24 \\ & (0.2) \end{aligned}$ | $\begin{aligned} & -0.23 \\ & (0.22) \end{aligned}$ | $\begin{gathered} 0.3 \\ (0.2) \end{gathered}$ |
| COOP | PM | $\begin{gathered} 0.2 \\ (0.3) \end{gathered}$ | $\begin{gathered} 0.26 \\ (0.13) \end{gathered}$ | $\begin{gathered} 0.42 \\ (0.15) \end{gathered}$ | $\begin{gathered} 0.23 \\ (0.18) \end{gathered}$ | $\begin{aligned} & -0.35 \\ & (0.13) \end{aligned}$ |
| PM | SOH | $\begin{aligned} & -0.33 \\ & (2.86) \end{aligned}$ | $\begin{gathered} 0.95 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.08 \\ & (0.19) \end{aligned}$ | $\begin{aligned} & -0.26 \\ & (0.2) \end{aligned}$ | $\begin{gathered} 0.08 \\ (0.15) \end{gathered}$ |
| SOH | PM | $\begin{aligned} & -13.06 \\ & (13.0) \end{aligned}$ | $\begin{gathered} 1.2 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.15 \\ & (0.22) \end{aligned}$ | $\begin{aligned} & -0.25 \\ & (0.2) \end{aligned}$ | $\begin{array}{r} 0.05 \\ (0.1) \end{array}$ |
| PM | SMH | $\begin{aligned} & -2.5 \\ & (3.2) \end{aligned}$ | $\begin{gathered} 0.95 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.23 \\ & (0.19) \end{aligned}$ | $\begin{aligned} & -0.004 \\ & (0.19) \end{aligned}$ | $\begin{aligned} & -0.09 \\ & (0.14) \end{aligned}$ |
| SMH | PM | $\begin{gathered} 5.9 \\ (5.8) \end{gathered}$ | $\begin{gathered} 0.96 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.27 \\ & (0.21) \end{aligned}$ | $\begin{array}{r} 0.19 \\ (0.2) \end{array}$ | $\begin{aligned} & -0.16 \\ & (0.13) \end{aligned}$ |
| PM | TB | $\begin{gathered} 8.4 \\ (12.5) \end{gathered}$ | $\begin{gathered} 0.9 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.25 \\ & (0.19) \end{aligned}$ | $\begin{array}{r} 0.005 \\ (0.19) \end{array}$ | $\begin{aligned} & -0.02 \\ & (0.14) \end{aligned}$ |
| TB | PM | $\begin{gathered} 0.61 \\ (0.26) \end{gathered}$ | $\begin{gathered} 1.22 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.5 \\ & (0.22) \end{aligned}$ | $\begin{gathered} 0.41 \\ (0.2) \end{gathered}$ | $\begin{aligned} & -0.25 \\ & (0.13) \end{aligned}$ |
| PM | FIMP | $\begin{array}{r} -2.15 \\ (3.4) \end{array}$ | $\begin{gathered} 0.76 \\ (0.29) \end{gathered}$ | $\begin{aligned} & -0.12 \\ & (0.32) \end{aligned}$ | $\begin{aligned} & -0.39 \\ & (0.32) \end{aligned}$ | $\begin{gathered} 0.38 \\ (0.27) \end{gathered}$ |
| FIMP | PM | $\begin{gathered} 1.45 \\ (5.01) \end{gathered}$ | $\begin{gathered} 0.78 \\ (0.27) \end{gathered}$ | $\begin{aligned} & -0.42 \\ & (0.31) \end{aligned}$ | $\begin{gathered} 0.68 \\ (0.3) \end{gathered}$ | $\begin{aligned} & -0.71 \\ & (0.26) \end{aligned}$ |
| PO | CORPF | $\begin{aligned} & -0.03 \\ & (0.39) \end{aligned}$ | $\begin{gathered} 0.59 \\ (0.22) \end{gathered}$ | $\begin{gathered} 0.51 \\ (0.28) \end{gathered}$ | $\begin{aligned} & -0.78 \\ & (0.29) \end{aligned}$ | $\begin{gathered} 0.27 \\ (0.22) \end{gathered}$ |
| CORPF | PO | $\begin{aligned} & -0.01 \\ & (0.02) \end{aligned}$ | $\begin{gathered} 1.30 \\ (0.23) \end{gathered}$ | $\begin{aligned} & -0.85 \\ & (0.3) \end{aligned}$ | $\begin{gathered} 0.98 \\ (0.31) \end{gathered}$ | $\begin{aligned} & -0.85 \\ & (0.22) \end{aligned}$ |
| po | SBX | $\begin{gathered} 0.2 \\ (0.58) \end{gathered}$ | $\begin{gathered} 0.75 \\ (0.15) \end{gathered}$ | $\begin{gathered} 0.30 \\ (0.19) \end{gathered}$ | $\begin{aligned} & -0.09 \\ & (0.19) \end{aligned}$ | $\begin{aligned} & -0.14 \\ & (0.15) \end{aligned}$ |


| $\beta(1)$ | $\beta(2)$ | $\beta(3)$ | $\beta(4)$ | $\beta(5)$ | $\beta(6)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| -3.45 | 2.92 | -0.83 | -0.97 | -0.09 | 1.38 |
| $(1.26)$ | $(1.36)$ | $(1.31)$ | $(1.26)$ | $(1.19)$ | $(1.12)$ |
| 0.06 | -0.05 | 0.03 | 0.01 | 0.03 | -0.01 |
| $(0.01)$ | $(0.02)$ | $(0.03)$ | $(0.02)$ | $(0.03)$ | $(0.03)$ |
| 0.10 | -0.13 | 0.03 | 0.05 | -0.06 | 0.04 |
| $(0.03)$ | $(0.05)$ | $(0.05)$ | $(0.05)$ | $(0.05)$ | $(0.03)$ |
|  |  |  |  |  |  |
| -1.6 | 1.84 | -1.46 | 1.73 | -1.3 | 0.19 |
| $(0.6)$ | $(0.8)$ | $(0.84)$ | $(0.8)$ | $(0.9)$ | $(0.7)$ |
| 0.07 | -0.07 | 0.08 | -0.13 | 0.03 | 0.04 |
| $(0.08)$ | $(0.12)$ | $(0.11)$ | $(0.11)$ | $(0.12)$ | $(0.08)$ |
| 0.02 | -0.01 | 0.26 | 0.12 | 0.07 | 0.16 |
| $(0.2)$ | $(0.3)$ | $(0.3)$ | $(0.3)$ | $(0.4)$ | $(0.4)$ |
| 1.08 | 5.26 | -11.69 | 4.3 | -7.6 | 6.5 |
| $(6.7)$ | $(10.6)$ | $(11.12)$ | $(11.1)$ | $(10.6)$ | $(6.6)$ |
| 0.005 | -0.006 | 0.005 | -0.003 | 0.006 | -0.007 |
| $(0.003)$ | $(0.004)$ | $(0.004)$ | $(0.004)$ | $(0.005)$ | $(0.004)$ |
| 0.15 | -0.1 | 0.24 | -0.34 | -0.02 | 0.09 |
| $(0.2)$ | $(0.2)$ | $(0.2)$ | $(0.2)$ | $(0.15)$ | $(0.13)$ |
| 0.39 | 0.48 | -0.90 | 0.77 | -0.19 | 0.19 |
| $(0.42)$ | $(0.46)$ | $(0.47)$ | $(0.43)$ | $(0.34)$ | $(0.35)$ |
| 5.28 | -8.79 | 13.14 | -3.89 | -5.12 | 0.37 |
| $(3.63)$ | $(5.19)$ | $(5.16)$ | $(4.25)$ | $(3.35)$ | $(2.7)$ |
| -0.02 | 0.04 | -0.05 | 0.04 | -0.01 | 0.004 |
| $(0.01)$ | $(0.02)$ | $(0.02)$ | $(0.01)$ | $(0.01)$ | $(0.01)$ |
| 0.05 | 0.02 | -0.02 | -0.02 | -0.01 | 0.02 |
| $(0.02)$ | $(0.02)$ | $(0.03)$ | $(0.03)$ | $(0.03)$ | $(0.03)$ |
|  |  |  |  |  |  |

Table B. 4 - continued

| Model: | $\begin{gathered} Y(t)=c+\sum_{s=1}^{4} \alpha(s) Y(t-s)+ \\ \quad \gamma_{3} S p r i+e_{t} \end{gathered}$ |  | $\sum_{k=1}^{6} \beta(k) X(t-k)+\gamma_{1} \text { Fall }+\gamma_{2} \text { Wint }+$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Fall | Wint | Spri | $\mathrm{F}_{6,50}$ | $\mathrm{R}^{2}$ | MSE |
| $\begin{aligned} & -7.4 \\ & (4.9) \end{aligned}$ | $\begin{gathered} 3.8 \\ (5.14) \end{gathered}$ | $\begin{gathered} 5.58 \\ (5.14) \end{gathered}$ | $2.23 *$ | 0.69 | 503.6 |
| $\frac{-1.12}{(0.6)}$ | $\begin{gathered} 0.42 \\ (0.61) \end{gathered}$ | $\begin{gathered} 0.58 \\ (0.57) \end{gathered}$ | 5.4* | 0.80 | 6.5 |
| $\begin{aligned} & -3.3 \\ & (7.6) \end{aligned}$ | $\begin{aligned} & -6.5 \\ & (7.5) \end{aligned}$ | $\begin{gathered} 5.8 \\ (6.9) \end{gathered}$ | 2.83* | 0.71 | 476. |
| $\begin{aligned} & 137.5 \\ & (27.9) \end{aligned}$ | $\begin{array}{r} 5.07 \\ (32.9) \end{array}$ | $\begin{aligned} & -40.4 \\ & (30.0) \end{aligned}$ | 2.2** | 0.84 | 9,578. |
| $\begin{aligned} & -5.6 \\ & (7.66) \end{aligned}$ | $\begin{gathered} 4.3 \\ (6.7) \end{gathered}$ | $\begin{gathered} 6.4 \\ (6.6) \end{gathered}$ | 0.40 | 0.6 | 609. |
| $\begin{gathered} 16.9 \\ (11.6) \end{gathered}$ | $\begin{gathered} 7.7 \\ (12.2) \end{gathered}$ | $\begin{gathered} 10.5 \\ (11.3) \end{gathered}$ | 1.36 | 0.82 | 1,882. |
| $\begin{aligned} & -4.9 \\ & (6.05) \end{aligned}$ | $\begin{gathered} 1.82 \\ (5.8) \end{gathered}$ | $\begin{gathered} 3.5 \\ (5.8) \end{gathered}$ | 0.65 | 0.64 | 592.7 |
| $\begin{aligned} & -0.09 \\ & (0.12) \end{aligned}$ | $\begin{aligned} & -0.05 \\ & (0.13) \end{aligned}$ | $\begin{aligned} & -0.17 \\ & (0.12) \end{aligned}$ | 1.17 | 0.90 | 0.25 |
| $\begin{aligned} & -4.6 \\ & (5.8) \end{aligned}$ | $\begin{gathered} 6.47 \\ (5.87) \end{gathered}$ | $\begin{array}{r} 3.77 \\ (5.8) \end{array}$ | 0.70 | 0.65 | 588.8 |
| $\begin{gathered} 2.71 \\ (8.61) \end{gathered}$ | $\begin{gathered} 9.63 \\ (8.73) \end{gathered}$ | $\begin{array}{r} 3.76 \\ (8.4) \end{array}$ | 1.39 | 0.80 | 1.245. |
| $\begin{aligned} & -0.62 \\ & (0.77) \end{aligned}$ | $\begin{aligned} & -0.38 \\ & (0.8) \end{aligned}$ | $\begin{aligned} & -0.42 \\ & (0.76) \end{aligned}$ | 2.46* | 0.78 | 9.86 |
| $\begin{aligned} & -0.09 \\ & (0.04) \end{aligned}$ | $\begin{gathered} 0.05 \\ (0.04) \end{gathered}$ | $\begin{aligned} & -0.02 \\ & (0.04) \end{aligned}$ | 2.32 * | 0.86 | 0.03 |
| $\begin{gathered} 0.76 \\ (1.11) \end{gathered}$ | $\begin{gathered} 0.42 \\ (1.17) \end{gathered}$ | $\begin{gathered} -1.39 \\ (1.1) \end{gathered}$ | 1.51 | 0.76 | 10.8 |

Table B. 4 - continued

| Mode1: | $\begin{aligned} Y(t)= & C+\sum_{s=1}^{4} \alpha(s) Y(t-s) \\ & \gamma_{3} \operatorname{Spri}+e_{t} \end{aligned}$ |  | $\sum_{k=1}^{6} \beta(k) X(t-k)+\gamma_{1} \text { Fall }+\gamma_{2} \text { Wint }+$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | X | $\begin{gathered} C \\ (s \cdot e) \end{gathered}$ | $\alpha(1)$ | $\alpha(2)$ | $\alpha(3)$ | $\alpha(4)$ |
| SBX | PO | $\begin{aligned} & -7.7 \\ & (3.3) \end{aligned}$ | $\begin{gathered} 0.07 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.09 \\ & (0.15) \end{aligned}$ | $\begin{aligned} & -0.25 \\ & (0.15) \end{aligned}$ | $\begin{gathered} 0.37 \\ (0.15) \end{gathered}$ |
| PO | HPAU | $\begin{gathered} 0.15 \\ (0.45) \end{gathered}$ | $\begin{gathered} 0.84 \\ (0.14) \end{gathered}$ | $\begin{gathered} 0.09 \\ (0.18) \end{gathered}$ | $\begin{gathered} -0.007 \\ (0.09) \end{gathered}$ | $\begin{gathered} -0.03 \\ (0.15) \end{gathered}$ |
| HPAU | PO | $\begin{aligned} & -0.08 \\ & (0.08) \end{aligned}$ | $\begin{gathered} 0.69 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.13 \\ & (0.16) \end{aligned}$ | $\begin{gathered} 0.18 \\ (0.16) \end{gathered}$ | $\begin{gathered} 0.02 \\ (0.13) \end{gathered}$ |
| PO | COOP | $\begin{gathered} 0.04 \\ (0.39) \end{gathered}$ | $\begin{gathered} 0.78 \\ (0.17) \end{gathered}$ | $\begin{aligned} & -0.14 \\ & (0.2) \end{aligned}$ | $\begin{gathered} -0.22 \\ (0.2) \end{gathered}$ | $\begin{array}{r} 0.55 \\ (0.2) \end{array}$ |
| COOP | PO | $\begin{gathered} 0.15 \\ (0.33) \end{gathered}$ | $\begin{gathered} 0.08 \\ (0.18) \end{gathered}$ | $\begin{gathered} 0.44 \\ (0.17) \end{gathered}$ | $\begin{gathered} 0.02 \\ (0.18) \end{gathered}$ | $\begin{aligned} & -0.58 \\ & (0.17) \end{aligned}$ |
| PO | SOH | $\begin{array}{r} 0.002 \\ (0.42) \end{array}$ | $\begin{gathered} 0.89 \\ (0.14) \end{gathered}$ | $\begin{gathered} 0.2 \\ (0.19) \end{gathered}$ | $\begin{aligned} & -0.26 \\ & (0.19) \end{aligned}$ | $\begin{aligned} & -0.03 \\ & (0.15) \end{aligned}$ |
| SOH | PO | $\begin{aligned} & -14.6 \\ & (12.8) \end{aligned}$ | $\begin{gathered} 0.99 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.11 \\ & (0.2) \end{aligned}$ | $\begin{aligned} & -0.11 \\ & (0.2) \end{aligned}$ | $\begin{aligned} & -0.002 \\ & (0.13) \end{aligned}$ |
| PO | SMH | $\begin{aligned} & -0.09 \\ & (0.48) \end{aligned}$ | $\begin{gathered} 0.79 \\ (0.14) \end{gathered}$ | $\begin{gathered} 0.07 \\ (0.19) \end{gathered}$ | $\begin{aligned} & -0.04 \\ & (0.09) \end{aligned}$ | $\begin{aligned} & -0.04 \\ & (0.16) \end{aligned}$ |
| SMH | PO | $\begin{aligned} & 10.3 \\ & (5.9) \end{aligned}$ | $\begin{gathered} 0.89 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.36 \\ & (0.19) \end{aligned}$ | $\begin{gathered} 0.23 \\ (0.19) \end{gathered}$ | $\begin{aligned} & -0.24 \\ & (0.14) \end{aligned}$ |
| PO | TB | $\begin{aligned} & -1.2 \\ & (1.8) \end{aligned}$ | $\begin{gathered} 0.68 \\ (0.14) \end{gathered}$ | $\begin{gathered} 0.11 \\ (0.17) \end{gathered}$ | $\begin{aligned} & -0.03 \\ & (0.18) \end{aligned}$ | $\begin{gathered} 0.01 \\ (0.14) \end{gathered}$ |
| TB | PO | $\begin{gathered} 0.5 \\ (0.28) \end{gathered}$ | $\begin{gathered} 1.3 \\ (0.14) \end{gathered}$ | $\begin{gathered} -0.54 \\ (0.2) \end{gathered}$ | $\begin{array}{r} 0.44 \\ (0.2) \end{array}$ | $\begin{aligned} & -0.30 \\ & (0.14) \end{aligned}$ |
| PO | FIMP | $\begin{aligned} & -0.22 \\ & (0.36) \end{aligned}$ | $\begin{aligned} & 0.4 \\ & (0.17) \end{aligned}$ | $\begin{array}{r} 0.07 \\ (0.2) \end{array}$ | $\begin{aligned} & -0.29 \\ & (0.2) \end{aligned}$ | $\begin{gathered} 0.18 \\ (0.14) \end{gathered}$ |
| FIMP | P0 | $\begin{aligned} & -1.76 \\ & (4.51) \end{aligned}$ | $\begin{gathered} 1.19 \\ (0.17) \end{gathered}$ | $\begin{aligned} & -0.53 \\ & (0.25) \end{aligned}$ | $\begin{gathered} 0.55 \\ (0.25) \end{gathered}$ | $\begin{aligned} & -0.42 \\ & (0.19) \end{aligned}$ |


| $\beta(1)$ | $\beta(2)$ | $\beta(3)$ | $\beta(4)$ | $\beta(5)$ | $\beta(6)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| -0.14 | 0.34 | -1.59 | 0.28 | 0.4 | -0.22 |
| $(0.83)$ | $(1.05)$ | $(1.05)$ | $(1.05)$ | $(1.05)$ | $(0.85)$ |
| 1.22 | -0.43 | 0.77 | -0.61 | 0.28 | -0.43 |
| $(0.78)$ | $(0.95)$ | $(0.93)$ | $(0.93)$ | $(0.91)$ | $(0.72)$ |
| -0.005 | -0.03 | 0.007 | -0.01 | -0.006 | 0.018 |
| $(0.02)$ | $(0.03)$ | $(0.03)$ | $(0.03)$ | $(0.03)$ | $(0.03)$ |
| 0.02 | 0.57 | -0.16 | -0.61 | -0.11 | -0.01 |
| $(0.21)$ | $(0.2)$ | $(0.2)$ | $(0.2)$ | $(0.2)$ | $(0.15)$ |
| 0.73 | -0.34 | -0.04 | 0.27 | 0.04 | 0.11 |
| $(0.16)$ | $(0.18)$ | $(0.19)$ | $(0.18)$ | $(0.17)$ | $(0.14)$ |
| 0.01 | -0.01 | 0.0002 | 0.01 | -0.01 | 0.005 |
| $(0.004)$ | $(0.006)$ | $(0.007)$ | $(0.007)$ | $(0.007)$ | $(0.005)$ |
| -3.73 | -0.23 | -4.2 | 6.7 | -4.06 | 7.84 |
| $(4.3)$ | $(5.6)$ | $(5.3)$ | $(5.3)$ | $(5.6)$ | $(5.02)$ |
| 0.02 | -0.01 | -0.001 | -0.01 | 0.01 | -0.003 |
| $(0.01)$ | $(0.02)$ | $(0.02)$ | $(0.02)$ | $(0.02)$ | $(0.01)$ |
| 6.07 | -0.89 | -0.55 | -0.86 | -0.96 | 4.06 |
| $(1.75)$ | $(2.2)$ | $(2.2)$ | $(2.2)$ | $(2.3)$ | $(1.9)$ |
| 0.91 | -0.14 | 0.32 | 0.29 | -0.4 | -0.79 |
| $(0.94)$ | $(1.49)$ | $(1.49)$ | $(1.48)$ | $(1.42)$ | $(0.88)$ |
| -0.002 | -0.05 | 0.05 | -0.01 | 0.02 | -0.02 |
| $(0.02)$ | $(0.03)$ | $(0.03)$ | $(0.03)$ | $(0.03)$ | $(0.02)$ |
| 0.01 | 0.002 | 0.03 | 0.001 | -0.01 | 0.03 |
| $(0.01)$ | $(0.02)$ | $(0.02)$ | $(0.01)$ | $(0.01)$ | $(0.01)$ |
| -3.77 | 4.4 | 04.5 | 3.5 | 00.3 | 0.15 |
| $(2.14)$ | $(2.5)$ | $(2.5)$ | $(2.3)$ | $(2.04)$ | $(1.69)$ |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

Table B. 4 - continued

| Mode1: <br> Fall | $\begin{aligned} Y(t)= & C+\sum_{s=1}^{4} \alpha(s) Y(t-s) \\ & \gamma_{3} S p r i+e_{t} \end{aligned}$ |  | $\sum_{k=1}^{6} \beta(k) X(t-k)+\gamma_{1} \text { Fall }+\gamma_{2} \text { Wint }+$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Wint | Spri | $\mathrm{F}_{6,50}$ | $\mathrm{R}^{2}$ | MSE |
| $\begin{aligned} & 12.3 \\ & (6.01) \end{aligned}$ | $\begin{gathered} 2.26 \\ (6.12) \end{gathered}$ | $\begin{gathered} 1.45 \\ (6.17) \end{gathered}$ | 1.05 | 0.57 | 350.1 |
| $\begin{aligned} & -0.56 \\ & (3.65) \end{aligned}$ | $\begin{array}{r} -2.75 \\ \quad(4.05) \end{array}$ | $\begin{array}{r} 0.34 \\ (3.7) \end{array}$ | 0.79 | 0.74 | 11.6 |
| $\begin{gathered} 2.93 \\ (0.56) \end{gathered}$ | $\begin{aligned} & -0.79 \\ & (0.64) \end{aligned}$ | $\begin{aligned} & -2.67 \\ & (0.57) \end{aligned}$ | 1.44 | 0.94 | 0.38 |
| $\begin{gathered} -1.39 \\ (0.7) \end{gathered}$ | $\begin{aligned} & -0.04 \\ & (0.7) \end{aligned}$ | $\begin{array}{r} 0.95 \\ (0.7) \end{array}$ | 2.96* | 0.79 | 9.4 |
| $\begin{aligned} & -1.69 \\ & (0.62) \end{aligned}$ | $\begin{gathered} 0.22 \\ (0.62) \end{gathered}$ | $\begin{gathered} 1.31 \\ (0.62) \end{gathered}$ | 5.03* | 0.80 | 6.69 |
| $\begin{aligned} & -0.07 \\ & (1.24) \end{aligned}$ | $\begin{aligned} & -1.74 \\ & (1.13) \end{aligned}$ | $\begin{gathered} 0.15 \\ (1.09) \end{gathered}$ | 1.82 | 0.77 | 10.5 |
| $\begin{aligned} & 113.4 \\ & (28.3) \end{aligned}$ | $\begin{array}{r} 15.76 \\ (30.9) \end{array}$ | $\begin{aligned} & -19.8 \\ & (31.1) \end{aligned}$ | $2.34 *$ | 0.84 | 9,454. |
| $\begin{gathered} 0.19 \\ (1.06) \end{gathered}$ | $\begin{aligned} & -0.3 \\ & (0.94) \end{aligned}$ | $\begin{aligned} & -0.26 \\ & (0.92) \end{aligned}$ | 0.73 | 0.74 | 11.7 |
| $\begin{gathered} 14.8 \\ (10.7) \end{gathered}$ | $\begin{gathered} 4.7 \\ (10.9) \end{gathered}$ | $\begin{array}{r} 14.57 \\ (10.7) \end{array}$ | 2.88* | 0.84 | 1,627. |
| $\begin{aligned} & -0.87 \\ & (0.8) \end{aligned}$ | $\begin{aligned} & -0.13 \\ & (0.8) \end{aligned}$ | $\begin{array}{r} 0.15 \\ (0.8) \end{array}$ | 1.4 | 0.76 | 10.9 |
| $\begin{aligned} & -0.11 \\ & (0.12) \end{aligned}$ | $\begin{aligned} & -0.06 \\ & (0.12) \end{aligned}$ | $\begin{aligned} & -0.17 \\ & (0.12) \end{aligned}$ | 1.25 | 0.9 | 0.25 |
| $\begin{aligned} & -0.56 \\ & (0.64) \end{aligned}$ | $\begin{aligned} & -0.29 \\ & (0.67) \end{aligned}$ | $\begin{gathered} 0.28 \\ (0.66) \end{gathered}$ | 5.37* | 0.83 | 7.76 |
| $\begin{gathered} 8.2 \\ (8.3) \end{gathered}$ | $\begin{gathered} -0.17 \\ (8.5) \end{gathered}$ | $\begin{array}{r} 2.23 \\ (8.5) \end{array}$ | 1.33 | 0.80 | 1,254. |

Table B. 4 - continued


| $\beta$ (1) | $\beta$ (2) | $\beta$ (3) | $\beta$ (4) | $\beta$ (5) | $\beta$ (6) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & -0.65 \\ & (5.6) \end{aligned}$ | $\begin{aligned} & -4.8 \\ & (5.6) \end{aligned}$ | $\begin{aligned} & -1.09 \\ & (2.9) \end{aligned}$ | $\begin{aligned} & -0.44 \\ & (2.96) \end{aligned}$ | $\begin{aligned} & -1.65 \\ & (5.68) \end{aligned}$ | $\begin{gathered} 3.19 \\ (5.70) \end{gathered}$ |
| 0.001 | 0.0002 | -0.0005 | 0.007 | -0.003 | 0.0006 |
| $\begin{aligned} & -0.16 \\ & (0.13) \end{aligned}$ | $\begin{gathered} 0.05 \\ (0.17) \end{gathered}$ | $\begin{aligned} & -0.1 \\ & (0.16) \end{aligned}$ | $\begin{gathered} 0.05 \\ (0.16) \end{gathered}$ | $\begin{gathered} 0.06 \\ (0.16) \end{gathered}$ | $\begin{gathered} 0.12 \\ (0.13) \end{gathered}$ |
| 0.06 | 0.13 | -0.13 | 0.04 | 0.02 | -0.04 |
| $\begin{aligned} & -0.005 \\ & (0.03) \end{aligned}$ | $\begin{array}{r} 0.005 \\ (0.03) \end{array}$ | $\begin{aligned} & 0.0003 \\ & (0.03) \end{aligned}$ | $\begin{gathered} 0.01 \\ (0.03) \end{gathered}$ | $\begin{aligned} & -0.05 \\ & (0.03) \end{aligned}$ | $\begin{gathered} 0.05 \\ (0.02) \end{gathered}$ |
| $\begin{gathered} 1.77 \\ (0.75) \end{gathered}$ | $\begin{aligned} & -1.05 \\ & (0.94) \end{aligned}$ | $\begin{aligned} & -0.05 \\ & (0.98) \end{aligned}$ | $\begin{gathered} 0.45 \\ (0.99) \end{gathered}$ | $\begin{gathered} 0.34 \\ (1.14) \end{gathered}$ | $\begin{aligned} & -0.83 \\ & (0.89) \end{aligned}$ |
| $\begin{aligned} & -11.56 \\ & (12.9) \end{aligned}$ | $\begin{gathered} 2.21 \\ (16.72) \end{gathered}$ | $\begin{gathered} 14.35 \\ (16.87) \end{gathered}$ | $\begin{gathered} -5.49 \\ (17.07) \end{gathered}$ | $\begin{gathered} 6.76 \\ (17.04) \end{gathered}$ | $\begin{aligned} & -15.3 \\ & (14.22) \end{aligned}$ |
| $\begin{gathered} 0.0004 \\ (0.0015) \end{gathered}$ | $\begin{aligned} & -0.0004 \\ & (0.0018) \end{aligned}$ | $\begin{aligned} & -0.0008 \\ & (0.0018) \end{aligned}$ | $\begin{aligned} & -0.0002 \\ & (0.0018) \end{aligned}$ | $\begin{gathered} 0.0009 \\ (0.0018) \end{gathered}$ | $\begin{aligned} & -0.001 \\ & (0.001) \end{aligned}$ |
| $\begin{aligned} & -0.18 \\ & (0.43) \end{aligned}$ | $\begin{aligned} & -0.45 \\ & (0.59) \end{aligned}$ | $\begin{gathered} 0.71 \\ (0.59) \end{gathered}$ | $\begin{gathered} 0.13 \\ (0.60) \end{gathered}$ | $\begin{gathered} 0.57 \\ (0.68) \end{gathered}$ | $\begin{aligned} & -0.9 \\ & (0.54) \end{aligned}$ |
| $\begin{array}{r} 0.001 \\ .0 .04) \end{array}$ | $\begin{aligned} & -0.03 \\ & (0.05) \end{aligned}$ | $\begin{aligned} & -0.005 \\ & (0.05) \end{aligned}$ | $\begin{gathered} 0.02 \\ (0.05) \end{gathered}$ | $\begin{aligned} & -0.008 \\ & (0.05) \end{aligned}$ | $\begin{aligned} & -0.003 \\ & (0.04) \end{aligned}$ |
| $\begin{gathered} 2.25 \\ (3.38) \end{gathered}$ | $\begin{gathered} 0.84 \\ (4.06) \end{gathered}$ | $\begin{aligned} & -1.19 \\ & (4.06) \end{aligned}$ | $\begin{aligned} & -5.02 \\ & (4.08) \end{aligned}$ | $\begin{aligned} & -0.64 \\ & (4.26) \end{aligned}$ | $\begin{gathered} 1.56 \\ (3.65) \end{gathered}$ |
| $\begin{aligned} & -0.004 \\ & (0.006) \end{aligned}$ | $\begin{aligned} & -0.0002 \\ & (0.007) \end{aligned}$ | $\begin{aligned} & -0.009 \\ & (0.007) \end{aligned}$ | $\begin{aligned} & -0.001 \\ & (0.007) \end{aligned}$ | $\begin{gathered} 0.004 \\ (0.007) \end{gathered}$ | $\begin{aligned} & -0.003 \\ & (0.006) \end{aligned}$ |

Table B. 4 - continued


Table B.5. Autoregressive model of the detrend futures prices of PS, PO and PM

| Mode1: | $Y(t)=C+\sum_{s=1}^{6} \alpha(s) Y(t-s)+\sum_{i=1}^{3} \gamma_{i} D_{i t}+e_{t},$ <br> $D_{\text {it }}$ 's are seasonal dummy variables |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
|  | C | $\alpha$ (1) | $\alpha(2)$ | $\alpha$ (3) | $\alpha$ (4) | $\alpha$ (5) | $\alpha(6)$ |
| $\mathrm{PSF}_{\mathrm{t}}$ | $\begin{aligned} & -0.03 \\ & (0.06) \end{aligned}$ | $\begin{gathered} 0.82 \\ (0.07)^{\mathrm{a}} \end{gathered}$ |  |  |  |  |  |
| $\mathrm{PSF}_{\mathrm{t}}$ | $\begin{aligned} & -0.03 \\ & (0.07) \end{aligned}$ | $\begin{gathered} 0.88 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.07 \\ & (0.13) \end{aligned}$ |  |  |  |  |
| $\mathrm{PSF}_{\mathrm{t}}$ | $\begin{aligned} & -0.03 \\ & (0.07) \end{aligned}$ | $\begin{gathered} 0.89 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.16 \\ & (0.17) \end{aligned}$ | $\begin{gathered} 0.10 \\ (0.13) \end{gathered}$ |  |  |  |
| $\mathrm{PSF}_{t}$ | $\begin{aligned} & -0.03 \\ & (0.07) \end{aligned}$ | $\begin{gathered} 0.89 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.16 \\ & (0.18) \end{aligned}$ | $\begin{gathered} 0.10 \\ (0.18) \end{gathered}$ | $\begin{aligned} & -0.001 \\ & (0.13) \end{aligned}$ |  |  |
| $\mathrm{PSF}_{\mathrm{t}}$ | $\begin{aligned} & -0.03 \\ & (0.07) \end{aligned}$ | $\begin{gathered} 0.81 \\ (0.09) \end{gathered}$ |  |  | $\begin{gathered} 0.004 \\ (0.14) \end{gathered}$ | $\begin{gathered} 0.02 \\ (0.13) \end{gathered}$ |  |
| $\mathrm{PSF}_{\mathrm{t}}$ | $\begin{aligned} & -0.03 \\ & (0.07) \end{aligned}$ | $\begin{gathered} 0.88 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.10 \\ & (0.15) \end{aligned}$ |  | $\begin{gathered} 0.03 \\ (0.15) \end{gathered}$ | $\begin{gathered} 0.19 \\ (0.18) \end{gathered}$ | $\begin{aligned} & -0.2 \\ & (0.14) \end{aligned}$ |
| $\mathrm{POF}_{\mathrm{t}}$ | $\begin{aligned} & -0.13 \\ & (0.35) \end{aligned}$ | $\begin{gathered} 0.82 \\ (0.07) \end{gathered}$ |  |  |  |  |  |
| $\mathrm{POF}_{\mathrm{t}}$ | $\begin{aligned} & -0.16 \\ & (0.34) \end{aligned}$ | $\begin{gathered} 1.04 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.27 \\ & (0.12) \end{aligned}$ |  |  |  |  |
| $\mathrm{POF}_{t}$ | $\begin{aligned} & -0.14 \\ & (0.33) \end{aligned}$ | $\begin{gathered} 1.11 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.54 \\ & (0.18) \end{aligned}$ | $\begin{gathered} 0.25 \\ (0.13) \end{gathered}$ |  |  |  |
| $\mathrm{POF}_{t}$ | $\begin{aligned} & -0.13 \\ & (0.32) \end{aligned}$ | $\begin{gathered} 1.19 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.7 \\ & (0.19) \end{aligned}$ | $\begin{gathered} 0.60 \\ (0.19) \end{gathered}$ | $\begin{aligned} & -0.13 \\ & (0.13) \end{aligned}$ |  |  |

${ }^{\text {a }}$ Number in parentheses is the standard deviation.
*Significant difference from zero at $\alpha=0.01$.

| $\gamma 1$ | $\gamma 2$ | $\gamma 3$ | MSE | $\mathrm{R}^{2}$ | F4,57 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & -0.06 \\ & (0.12) \end{aligned}$ | $\begin{aligned} & -0.16 \\ & (0.12) \end{aligned}$ | $\begin{aligned} & -0.03 \\ & (0.12) \end{aligned}$ | 0.30 | 0.69 | 32.38* |
| $\begin{aligned} & -0.07 \\ & (0.12) \end{aligned}$ | $\begin{aligned} & -0.15 \\ & (0.12) \end{aligned}$ | $\begin{aligned} & -0.02 \\ & (0.12) \end{aligned}$ | 0.31 | 0.69 | 25.6* |
| $\begin{aligned} & -0.07 \\ & (0.12) \end{aligned}$ | $\begin{aligned} & -0.12 \\ & (0.12) \end{aligned}$ | $\begin{aligned} & -0.03 \\ & (0.12) \end{aligned}$ | 0.31 | 0.69 | 21.32* |
| $\begin{aligned} & -0.07 \\ & (0.12) \end{aligned}$ | $\begin{aligned} & -0.12 \\ & (0.12) \end{aligned}$ | $\begin{aligned} & -0.03 \\ & (0.12) \end{aligned}$ | 0.31 | 0.69 | 17.95* |
| $\begin{aligned} & -0.06 \\ & (0.12) \end{aligned}$ | $\begin{aligned} & -0.16 \\ & (0.12) \end{aligned}$ | $\begin{aligned} & -0.03 \\ & (0.12) \end{aligned}$ | 0.31 | 0.68 | 20.90* |
| $\begin{aligned} & -0.15 \\ & (0.13) \end{aligned}$ | $\begin{aligned} & -0.13 \\ & (0.12) \end{aligned}$ | $\begin{gathered} 0.02 \\ (0.13) \end{gathered}$ | 0.31 | 0.70 | 16.14* |
| $\begin{array}{r} 0.63 \\ (0.6) \end{array}$ | $\begin{aligned} & -0.93 \\ & (0.6) \end{aligned}$ | $\begin{aligned} & -0.30 \\ & (0.6) \end{aligned}$ | 7.78 | 0.68 | 30.9* |
| $\begin{array}{r} 0.47 \\ (0.6) \end{array}$ | $\begin{aligned} & -1.06 \\ & (0.6) \end{aligned}$ | $\begin{aligned} & -0.03 \\ & (0.6) \end{aligned}$ | 7.3 | 0.7 | 27.15* |
| $\begin{gathered} 0.37 \\ (0.57) \end{gathered}$ | $\begin{aligned} & -0.94 \\ & (0.58) \end{aligned}$ | $\begin{array}{r} 0.17 \\ (0.6) \end{array}$ | 6.98 | 0.72 | 24.5* |
| $\begin{gathered} 0.61 \\ (0.56) \end{gathered}$ | $\begin{aligned} & -0.86 \\ & (0.55) \end{aligned}$ | $\begin{gathered} 0.09 \\ (0.57) \end{gathered}$ | 6.42 | 0.75 | 23.66* |

Table B. 5 - continued

| Model: $Y(t)=C+\sum_{s=1}^{6} \alpha(s) Y(t-s)+\sum_{i=1}^{3} \gamma_{i} D_{i t}+e_{t}$, <br> $\mathrm{D}_{\text {it }}$ 's are seasonal dummy variables |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  | C | $\alpha$ (1) | $\alpha$ (2) | $\alpha$ (3) | $\alpha$ (4) | $\alpha(5)$ | $\alpha$ (6) |
| $\mathrm{POF}_{t}$ | $\begin{aligned} & -0.15 \\ & (0.35) \end{aligned}$ | $\begin{gathered} 0.84 \\ (0.08) \end{gathered}$ |  |  | $\begin{gathered} 0.05 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.14 \\ & (0.13) \end{aligned}$ |  |
| $\mathrm{POF}_{\mathrm{t}}$ | $\begin{aligned} & -0.15 \\ & (0.35) \end{aligned}$ | $\begin{gathered} 1.03 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.26 \\ & (0.14) \end{aligned}$ |  | $\begin{gathered} 0.12 \\ (0.15) \end{gathered}$ | $\begin{aligned} & -0.21 \\ & (0.19) \end{aligned}$ | $\begin{gathered} 0.08 \\ (0.14) \end{gathered}$ |
| $\mathrm{PMF}_{\mathrm{t}}$ | $\begin{aligned} & -0.88 \\ & (1.86) \end{aligned}$ | $\begin{gathered} 0.79 \\ (0.07) \end{gathered}$ |  |  |  |  |  |
| $\mathrm{PMF}_{\mathrm{t}}$ | $\begin{aligned} & -0.88 \\ & (1.88) \end{aligned}$ | $\begin{gathered} 0.79 \\ (0.13) \end{gathered}$ | $\begin{aligned} & -0.004 \\ & (0.13) \end{aligned}$ |  |  |  |  |
| $\mathrm{PMF}_{\mathrm{t}}$ | $\begin{aligned} & -0.91 \\ & (1.86) \end{aligned}$ | $\begin{gathered} 0.79 \\ (0.13) \end{gathered}$ | $\begin{gathered} 0.15 \\ (0.17) \end{gathered}$ | $\begin{aligned} & -0.19 \\ & (0.13) \end{aligned}$ |  |  |  |
| $\mathrm{PMF}_{\mathrm{t}}$ | $\begin{aligned} & -0.92 \\ & (1.87) \end{aligned}$ | $\begin{gathered} 0.79 \\ (0.13) \end{gathered}$ | $\begin{gathered} 0.16 \\ (0.17) \end{gathered}$ | $\begin{gathered} -0.16 \\ (0.17) \end{gathered}$ | $\begin{aligned} & -0.03 \\ & (0.13) \end{aligned}$ |  |  |
| $\mathrm{PMF}_{\mathrm{t}}$ | $\begin{aligned} & -0.92 \\ & (1.88) \end{aligned}$ | $\begin{gathered} 0.83 \\ (0.08) \end{gathered}$ |  |  | $\begin{aligned} & -0.14 \\ & (0.14) \end{aligned}$ | $\begin{gathered} 0.06 \\ (0.13) \end{gathered}$ |  |
| $\mathrm{PMF}_{\mathrm{t}}$ | $\begin{gathered} -0.89 \\ (1.9) \end{gathered}$ | $\begin{gathered} 0.77 \\ (0.14) \end{gathered}$ | $\begin{gathered} 0.10 \\ (0.16) \end{gathered}$ |  | $\begin{aligned} & -0.18 \\ & (0.16) \end{aligned}$ | $\begin{gathered} 0.06 \\ (0.17) \end{gathered}$ | $\begin{gathered} 0.01 \\ (0.14) \end{gathered}$ |


| Y1 | $\gamma 2$ | $\gamma 3$ | MSE | $\mathrm{R}^{2}$ | $\mathrm{F}_{4,57}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 0.59 \\ (0.61) \end{gathered}$ | $\begin{aligned} & -0.81 \\ & (0.63) \end{aligned}$ | $\begin{aligned} & -0.29 \\ & (0.6) \end{aligned}$ | 7.79 | 0.68 | 20.90* |
| $\begin{gathered} 0.44 \\ (0.62) \end{gathered}$ | $\begin{aligned} & -0.85 \\ & (0.63) \end{aligned}$ | $\begin{aligned} & -0.09 \\ & (0.63) \end{aligned}$ | 7.56 | 0.71 | 16.63* |
| $\begin{aligned} & -2.65 \\ & (3.2) \end{aligned}$ | $\begin{aligned} & -2.16 \\ & (3.2) \end{aligned}$ | $\begin{aligned} & -2.22 \\ & (3.2) \end{aligned}$ | 219.4 | 0.63 | 25.67* |
| $\begin{aligned} & -2.68 \\ & (3.38) \end{aligned}$ | $\begin{aligned} & -2.15 \\ & (3.27) \end{aligned}$ | $\begin{aligned} & -2.21 \\ & (3.24) \end{aligned}$ | 223.17 | 0.63 | 20.19* |
| $\begin{aligned} & -2.32 \\ & (3.36) \end{aligned}$ | $\begin{aligned} & -3.55 \\ & (3.37) \end{aligned}$ | $\begin{aligned} & -1.51 \\ & (3.25) \end{aligned}$ | 218.7 | 0.65 | 17.53* |
| $\begin{aligned} & -2.19 \\ & (3.43) \end{aligned}$ | $\begin{aligned} & -3.52 \\ & (3.41) \end{aligned}$ | $\begin{aligned} & -1.77 \\ & (3.44) \end{aligned}$ | 222.38 | 0.65 | 14.79* |
| $\begin{aligned} & -2.91 \\ & (3.28) \end{aligned}$ | $\begin{aligned} & -2.46 \\ & (3.25) \end{aligned}$ | $\begin{aligned} & -2.6 \\ & (3.25) \end{aligned}$ | 222.6 | 0.64 | 17.06 |
| $\begin{aligned} & -2.15 \\ & (3.78) \end{aligned}$ | $\begin{aligned} & -2.98 \\ & (3.45) \end{aligned}$ | $\begin{aligned} & -2.98 \\ & (3.39) \end{aligned}$ | 229.09 | 0.64 | 12.48 |

APPENDIX C. SEASONAL TIME SERIES•

The explanation in this appendix is based upon McCleary and Hay, 1980.

Seasonality is a periodic movement of a time series that repeats itself at the same period. For quarterly time series such as in this research, seasonal pattern tends to repeat itself in every quarter of the crop year. It is debatable whether one should deseasonalize a time series. Nerlove (1964) suggestes that modelling seasonality according to causal behavior is the best way to handle seasonality. This view is also shared by Granger (1978).

Most of the agriculture product time series exhibit seasonal nonstationarity, that is the time series drift over time. For this type of series, one may take seasonal difference, for example:

$$
\begin{gathered}
\left(1-B^{4}\right) Y_{t}=e_{t} \\
\text { wisere } B^{4 /} Y_{t}=Y_{t-4} ; \text { or we can write: } \\
Y_{t}-Y_{t-4}=e_{t}
\end{gathered}
$$

However, for the stationary case, the coefficient of $Y_{t-4}$ is less than one as in the following:

$$
\begin{aligned}
& Y_{t}-\theta_{4} Y_{t-4}=e_{t} \text { or } \\
& \left(1-\theta_{4} B^{4}\right) Y_{t}=e_{. t}
\end{aligned}
$$

For higher-order seasonal patterns, we may have:

$$
\begin{aligned}
& Y_{t}-\theta_{4} Y_{t-4}-\theta_{8} Y_{t-8}=e_{t} \text { or } \\
& \left(1-\theta_{4} B^{4}-\theta_{8} B^{8}\right) Y_{t}=e_{t}
\end{aligned}
$$

Such higher order, however, is rare. In case of multiplicative seasonality, a time series may.exhibit the following:

$$
\begin{gathered}
\left(1-\phi_{1} B\right)\left(1-\phi_{4} B^{4}\right) Y_{t}=e_{t} \text { or } \\
\left(1-\phi_{1} B-\phi_{4} B^{4}+\phi_{1} \phi_{4} B^{5}\right) Y_{t}=e_{t}
\end{gathered}
$$

We can write $Y_{t}$ as:

$$
Y_{t}=\phi_{1} Y_{t-1}+\phi_{4} Y_{t-4}-\phi_{1} \phi_{4} Y_{t-5}+e_{t}
$$

The interact term $\phi_{1} \phi_{4}$ makes the analysis more complicated. For stationary time series, we require

$$
-1<\phi_{1} \text { and } \phi_{4}<+1
$$

Given the first-order of seasonal autoregressive:

$$
\left(1-\phi_{1} B\right)\left(1-\phi_{4} B^{4}\right) Y_{t}=e_{t}
$$

If the inverse of $\left(1-\phi_{1} B\right)$ and $\left(1-\phi_{4} B^{4}\right)$ exist, we can write them as:

$$
\begin{aligned}
Y_{t}= & \left(1-\phi_{1} B\right)^{-1}\left(1-\phi_{4} B^{4}\right)^{-1} e_{t} \text { or } \\
= & \left(1+\phi_{1} B+\phi_{1}^{2} B^{2}+\phi_{1}^{3} B^{3}+\ldots \phi_{1}^{n_{B} B^{n}}\right) \\
& *\left(1+\phi_{4} B^{4}+\phi_{4}^{2} B^{8}+\phi_{4}^{3} B^{24}+\ldots \phi_{4}^{n} B^{4 n}+\ldots\right) e_{t}
\end{aligned}
$$

The product of this series will converge if $\phi_{1}$ and $\phi_{4}$ satisfy the stationary condition.

APPENDIX D. DEFINITION OF VARIABLES
(capital letters refer to aggregate notations while small letter cases reserved for an agent notations)

| Notations | Description | Units |
| :---: | :---: | :---: |
| $\mathrm{A}_{\mathrm{t}}$ | Total U.S. soybean acreage planted at time $t$ | mil. acres |
| $\mathrm{C}_{1}$ | Cost of production of soybean | dollars |
| $\mathrm{C}_{2}$ | Adjustment cost of soybean acreage planted to soybeans | dollars |
| $\mathrm{C}_{3}$ | Farm cost of holding inventories | dollars |
| COOP | U.S. cotton seed oil, crude, tank cars, f.o.b. | c/lb |
| CORPF | U.S. corn price received by farmers | \$/bu. |
| CRNO | U.S. corn oil wholesale price | ¢/lb. |
| FIMP | Average wholesale price, Peruvian, imported | \$/ton |
| HPAU | U.S. quarterly high protein animal unit (computed) | 1000 units |
| $P_{t}$ | U.S. soybean price received by farmers | \$/bu. |
| PS ${ }_{t}$ | U.S. wholesale price of soybeans (Decatur) | \$/bu. |
| $\mathrm{PM}_{\mathrm{t}}$ | U.S. wholesale price of soybean meal, 44 percent protein, Decatur | \$/s.ton |
| $\mathrm{PO}_{t}$ | U.S. wholesale price of soybean oil, crude, Decatur | c/1b. |
| $\mathrm{SB}_{\mathrm{t}}$ | U.S. soybean production | mil. bu. |
| $\mathrm{SBB}_{t}$ | U.S. soybeans bought | mil. bu. |
| $S B C C_{t}$ | U.S. soybean commercial stocks | mil. bu. |
| $\mathrm{SBCM}_{t}$ | U.S. soybean crushing margin $\left(S B C M=\operatorname{SOMSC} * \frac{P M}{20}+\text { SOOSC } * P O-P S\right)$ | \$/bu. |
| $\mathrm{SBD}_{\mathrm{t}}$ | U.S. total demand for soybeans | mil. bu. |


| $\mathrm{SBHC}_{\mathrm{t}}$ | U.S. soybean stocks owned by Commodity Audit Cooperation | mil. bu. |
| :---: | :---: | :---: |
| $\mathrm{SBHF}_{\mathrm{t}}$ | U.S. soybean stocks on farms | mil. bu. |
| $\mathrm{SBS}_{\mathrm{t}}$ | U.S. soybeans sold by farmers | mil. bu. |
| SBX ${ }_{t}$ | U.S. soybean exports | mil. bu. |
| $\mathrm{SC}_{\mathrm{t}}$ | quantity of soybeans crushed at mills | mil. bu. |
| $\mathrm{SM}_{\mathrm{t}}$ | soybean meal production | 1000 s . ton |
| $\operatorname{SMDM}_{t}$ | U.S. total demand for soybean meal | 1000 s . ton |
| $\mathrm{SMH}_{\mathrm{t}}$ | U.S. total soybean meal stocks | 1000 s . ton |
| $\operatorname{six}_{t}$ | U.S. total soybean meal exports | 1000 s . ton |
| $\mathrm{SO}_{t}$ | U.S. soybean oil production | mil. 1 b . |
| SOMSC $_{\text {t }}$ | $\begin{aligned} & \text { Soybean meal crushing yield } \\ & \text { (SOMSC }=(0.02 * S M) / S C) \end{aligned}$ | cwt. per bu. |
| $\mathrm{SODM}_{\mathrm{t}}$ | U.S. total soybean demand | mil. 1 b . |
| $\mathrm{SOH}_{\mathrm{t}}$ | Soybean oil commercial stocks | mil. 1 l . |
| $\mathrm{SOHC}_{t}$ | Soybean oil stocks owned by CCC | mil. 1 lb . |
| $\mathrm{SoOSC}_{\mathrm{t}}$ | soybean oil crushing yield $(S O O S C=(0.01 * S O) / S C)$ | cwt. per bu |
| $\mathrm{sox}_{t}$ | U.S. total soybean oil commercial exports | mil. 1b. |
| TB | U.S. 90 days Treasury bill rates | \% |


[^0]:    2 Y fails to Granger cause $X$ if coefficient $b_{j}$ for all values of $j$ are statistically zero.

[^1]:    4 The figures " 50 " and " 100 " in (4.13) and (4.14) are ignored for the time being.

[^2]:    $1_{\text {Great }}$ appreciation is expressed to Duane Hacklander and George Allen, USDA, for their helpful suggestions concerning the computation of quarterly grain-consuming animal units and high-protein animal units.

[^3]:    ${ }^{a_{A l l}}$ variables in $A_{t}$ are multiplied by dummy variables zero one -- one in planting quarter and zero otherwise.
    ${ }^{\mathrm{b}}$ Standard deviation of the coefficient.
    *Significant at $\alpha=0.01$.

[^4]:    ${ }^{7} \mathrm{PS}_{\mathrm{t}}$ is a second-order autoregressive model; therefore, we need up to the first lag of $\mathrm{PS}_{t}$ in this equation.
    ${ }^{8}$ SBHC's are assumed fixed and given.

    $$
    { }^{9} \mathrm{SBX}_{t+2}=\mathrm{x}_{1} \mathrm{SBX} X_{t+1}+x_{2} S B X_{t}+x_{3} S B X_{t-1}+x_{4} S B X_{t-2} .
    $$

[^5]:    ${ }^{a_{A l l}}$ variables in soybean acreage equation are multiplied by the quarterly dummy variable zeroone - one in planting quarter and zero otherwise.
    $\mathrm{b}_{\text {Standard deviation. }}$

[^6]:    ${ }^{a_{S t a n d a r d ~}}$ deviation of the coefficient.
    *Significant at $\alpha=0.01$ or above.
    $* * S i g n i f i c a n t$ at $\alpha=0.05$ to $\alpha=0.1$.

